Regulatory Capital Requirements: Saving Too Much for Rainy Days?

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Abstract

Model risk needs to be recognized and accounted for in addition to market risk. Uncertainty in risk measures estimates may lead to false security in financial markets. We argue that quantile type risk-measures are at least as good as expected shortfall. We demonstrate how a bank can choose among competing models for measuring market risk and account for model risk. Some BCBS capital requirements formula currently in effect leads to excessive capital buffers even on an unstressed basis. We highlight that the loss to society associated with the inefficient minimum capital requirements calculations is economically substantial over time.

Keywords: Capital requirements, Model risk, Backtesting, Unexpected losses, Median Shortfall

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1 Motivation

FEDR (2011) mentions explicitly that "model risk should be managed like other types of risk and that banks should identify the sources of model risk and assess the magnitude". Similarly, EBA (2012) states: "Institutions should include the impact of valuation model risk when assessing the prudent value of its balance sheet. [..] Where possible an institution should quantify model risk by comparing the valuations produced from the full spectrum of modelling and calibration approaches." Nevertheless how to account for parameter estimation risk and how to adjust accordingly capital requirements is still very much open to research. Opaque internal models produce inconsistent risk valuations across the financial system. A more standardized approach, aimed at reducing the observed variability in risk calculations resulting from internal models of various banks relative to similar market positions, is desirable. On the other hand, banks point out that using a standardised approach may lead to increased systemic risk and inefficient capital usage that will ultimately lead to more problems down the line. Is it possible to emerge from this conundrum?

Model risk can be generally defined as the risk to incur a loss due to the implementation of potentially inaccurate financial models. Green and Figlewski (1999) provided evidence that imperfect models and inaccurate volatility forecasts may lead to significant risk exposure for banks. Significant errors in the estimation methods used in the finance industry were reported by Marshall and Siegel (1997), Berkowitz (2001) and Berkowitz and O' Brien (2002). Banks may also overestimate risk through VaR estimates (Pérignon et al., 2008). Estimation error in financial markets can be substantial, as demonstrated by Figlewski (2003) who offered a surprising but elegant explanation of extreme tail events that can be classified as rare only when distributions and their parameters are fully known. At the other end it is possible to have very different risk models that produce virtually identical performance under standard metrics, as highlighted by Chava et al. (2011). Even before the subprime crisis there was evidence, see Pérignon and Smith (2010), that more information about market risk was revealed to the public but its quality did not improve between 1996 and 2005. Here, we are trying to advocate for a more pro-active approach in accounting for parameter estimation uncertainty while at the same time consider also inference that gravitates towards a model-free or a general model approach. Enhancing risk estimation by calculating confidence intervals for VaR as detailed in this paper will allow risk managers and regulators to have a clear view on estimation error.

Within an ideal capital market the well-known theoretical argument of Modigliani and Miller implies that companies should not be concerned with risk management and capital allocation because investors can diversify their portfolios at no extra cost. However, due to asymmetric information, companies may find it expensive to increase their level of capitalization in difficult market conditions. Froot (2007) argued that even in normal times companies may miss the opportunity of engaging into profitable new projects when internal capital is low and external capital is expensive. Holding too much buffer capital is clearly costly but the regulator may insist on conservative calculations for capital requirements as a mechanism to control the risk of bank failure. Increasing capital requirements may trigger a sudden increase in shadow banking activities carrying much larger risk (Plantin, 2015). Excessive capital requirements reduce the tactical flexibility of banks expanding the problem of debt overhang and generate agency problems for the shareholders. Ibragimov et al. (2011) demonstrated that increasing capital requirements is detrimental also to the regulator since that capital cannot be used to offset negative externalities of systemic risk. Vallasca and Hagendorff (2013) revealed that the minimum capital requirements may not be robust, showing evidence of ill-calibration to a market measure of bank portfolio risk.

Our contribution in this paper unfolds as follows. First we present a critical comparison of the two main risk measures used in the risk management literature and finance industry, value-at-risk (VaR) and expected shortfall (ES), highlighting that from many theoretical and applied points of view, the former is to be preferred. Furthermore, we construct a rationale for employing the conditional Median Shortfall

(MS) as a measure of tail risk. The difference between MS and VaR is proposed here to represent *unexpected losses*, or losses beyond the maximum *expected* level measured by VaR. This is a new concept in risk management and we believe that risk forecasting calculations should be dichotomised into expected losses, that can be managed using VaR under normal market conditions, and unexpected losses defined as extra losses caused by market distress events. Then, since both VaR and MS are quantile based measures from a statistical point of view, we are able to use robust results from order statistics theory to design a tool able to validate and differentiate between competing models of forecasting market risk. To this end we are able to utilise the exact probability density or cumulative distribution function of our proposed quantile estimators of risk, under a wide range of distributions encountered in practice, and consequently we show how to calculate confidence intervals analytically or numerically.

While the literature has a myriad of statistical models to calculate VaR, in this paper we have a set of five most widely used models in risk management– nonparametric, gaussian, normal inverse gaussian (NIG), GARCH(1,1) and GJR-GARCH(1,1)–, and two main assets, one equity index and one foreign exchange. The upper bounds of the calculated confidence intervals are used to *adjust* the point estimates of market risk measures for parameter estimation uncertainty. Our methodology can be easily adapted to other distributions employed in the literature or in practice.

An exhaustive empirical exercise covering the period 02/01/1984 − 10/07/2014 allows us to judge the usefulness of these widely used methods, the new Basel 3 accord and also of our proposed method, by considering a battery of backtests. From our examples, the Lévy model based on the NIG distribution emerges as a flexible risk management model allowing calculations that are robust and superior to the other compared models. The last part of the paper is dedicated to the study of capital requirements calculations. We show that the BCBS recommended levels BCBS (2010), even when ignoring stressed market scenarios that occur rarely, are excessive for long periods of time. Moreover, capital requirements calculations using a model such as NIG adjusted for parameter estimation model risk is robust over time to market shocks and crashes that are endogenous to the market. In addition, we discuss the savings that could be generated by not locking vast amounts of capital as required by regulators.

Posner (2014) argued that it was always unclear how regulators precisely arrived at how high or low the minimum capital-asset ratio should be. In his study of U.S. regulators justifications for five regulations issued over more than 30 years Posner (2014) concluded that

> "regulators have never performed (or at least disclosed) a serious economic analysis that would justify the levels that they chose. Instead, regulators appear to have followed a practice of what I call "norming"-incremental change designed to weed out a handful of outlier banks. This approach resulted in a significant regulatory failure because it could not have given, and did not give, banks an adequate incentive to increase capital. The failure of banking regulators to use cost-benefit analysis in order to determine capital requirements may therefore have contributed to the financial crisis of 2007-2008".

The current state of academic and practitioner knowledge in risk management is still unclear as to which measure of risk and which model to use for calculating risk management buffers against future losses. Brock et al. (2003) and Brock et al. (2007) explore ways to integrate model risk into policy evaluation by model averaging methods. Our approach complies with recent specifications on how to deal with model risk. For example, EBA (2012) makes a distinction between the "prudent" value accounting for unexpected losses represented by the end point of a confidence interval generated by model risk and the fair value of a financial product. This is called the additional valuation adjustment (AVA). Our calculations of capital requirements adjusted for model risk are in the same unit as market risk and therefore, as argued by Detering and Packham (2015), from a regulatory point of view, systematic mispricing can be prevented and systemic risk can be reduced.

Our paper continues[1] the line of research showed in Kerkhof et al. (2010). Focusing on an equity index and an exchange rate, they proposed a method to explicitly include model risk, associated with the application of econometric methods, into the computation of the required levels of capital reserves[2]. While Kerkhof et al. (2010) capture model risk as a multiplication factor between a parametric gaussian model and the upper bound of a non-parametric model only, we advocate a wider more flexible methodology that can be used in a similar set-up. We show how one can obtain nonparametric and parametric confidence bounds specific to each distribution and moreover we identify a test for model validation focused on extreme losses. Moreover, we consider a much larger set of backtesting tests for VaR and ES and we extend our analysis over the subprime crisis period. Furthermore, we also analyse the MS as a tail risk measure competing with the ES and compare our capital requirements calculations with the values resulting from the BCBS formula. Our calculations of confidence intervals is more general and it does not rely on asymptotics statistics.

In our paper there are valuable lessons to learn both from a company perspective and a regulator perspective regarding which risk measure to use for market risk, how to account for parameter estimation risk for quantile based risk measures and how to validate models with a focus on tail risk. Furthermore, our approach for managing risk arrives at similar conclusions with Diamond and Rajan (2000), den Heuvel (2008) and Gorton and Winton (2014) that banking capital requirements may be economically and socially costly and that the banking system could be destabilized by following blindly regulatory minimums. We advocate a pro-active risk monitoring

[1]Other contributions proposed different solutions to quantify model risk across various asset classes, such as private equity investments (Bongaerts and Charlier, 2009), foreign exchange (Markiewicz, 2012), commodities (Barone-Adesi et al., 2016) or derivatives (Detering and Packham, 2015). Important papers covering various aspects of model risk in risk management area are Figlewski (2003), Cont et al. (2010), Kerkhof et al. (2010), Dowd (2010), Escanciano and Olmo (2011), Alexander and Sarabia (2012), Gourieroux and Zakoïan (2013), Boucher et al. (2014), Barone-Adesi (2015), Danielsson and Zhou (2015), Danielsson et al. (2016).

[2] Other papers aiming at determining capital requirements linked to model risk include Glasserman and Xu (2014) and Detering and Packham (2015).

approach based on models that are frequently backtested. This approach can save vast amounts of money that can be used elsewhere for the benefit of society.

The paper is structured as follows. In Section 2 we present a critique of VaR versus ES and we also compare them with a more recent measure called median shortfall. Section 3 is dedicated to model risk related to market risk, describing the main concepts, problems arising in this area and the technical solutions that we propose. Section 4 describes the data that is used in this study and Section 5 includes a suite of empirical applications demonstrating how to select the correct risk measure and risk model. Section 6 contains the results of backtesting based on several important tests. An important part of the paper is revealed in Section 7 that demonstrates how to incorporate model risk into capital requirements calculations and that important capital savings can be made that can be transferred elsewhere in the economy for the benefit of the society. The last section contains a summary of our results and recommendations based on our findings.

2 Which Risk Measure to Choose?

In this section we describe briefly a less known measure for tail risk (MS) and a critical comparative survey of the main properties of VaR, MS and ES highlighting that the pair VaR and MS will provide a risk management tool that is at least as good as ES.

2.1 Motivation for Conditional Median Shortfall

The VaR measure of market risk gives an indication of the maximum possible losses, at a given level of confidence, during normal market conditions. Hence, VaR is a measure of "expected losses". The ES on the other hand is a barometer of possible extreme, or rare losses. Thus, ES is a measure of "unexpected losses". There is a large literature on the pros and cons of each measure. Ideally one would like to capture both types of loss. Therefore, we advocate using two measures of risk, one for expected losses and associated with a higher order quantile and the other for unexpected losses and given by a measure of tail risk such as a low order quantile. There is a well documented positive and significant relationship between VaR as a measure of downside risk and the expected return on the market (Bali et al., 2009). Moreover, VaR at 99% confidence level was successfully employed by Allen et al. (2012) for estimating catastrophic risk in the financial sector. Working in a Lucas pure exchange economy Basak and Shapiro (2001) revealed that, when there are risk managers using VaR to control risk, the stock market volatility increases in down market and decreases in up markets.

For tail risk, ES has been the workhorse[3] in risk management for the last decade at least, its usage being strongly advocated since BCBS (2012). However, from a practical perspective, ES does have a major shortcoming in being very sensitive to outliers. Therefore, a very large loss triggered by rogue trading[4] will cause a distortion of the measure of tail risk and consequently an exacerbation of capital requirements that are driven by market risk. It is clearly impossible to forecast losses caused by rogue trading as intrinsic market risk losses[5]. Is it possible to find a measure of extreme tail risk that can capture endogenous tail market risk and that is better immunised to external market shocks?

We advocate in this paper using the *median shortfall* as a measure of tail risk. This is the same as minus the median of the conditional returns[6] lower than the VaR threshold as defined by So and Wong (2012). It was also suggested independently

[3]There were few measures introduced approximately at the same time to measure losses in the tail determined by VaR as a cutoff point. Acerbi and Tasche (2002) clarifying the differences between various measures. For our purposes, since we consider that the distribution function of the data generating process is always continuous, all these measures are identical.

[4]The losses in Barings case in 1995 reached $1.4 billion, for Amaranth in 2006 the loss was $3.295 billion, for Societe Generale in 2008 it was $7.2 billion and for UBS in 2011 it was reported as roughly $2 billion.

[5]Losses caused by rogue trading or other causes external to the market itself are still important but they need to be captured under operational risk not market risk.

[6]Since estimating VaR is a single-period ahead forecasting exercise, the calculations can be conducted in either returns space or loss and profit space. It is straightforward to recover the target quantile estimate of loss and profit from the quantile calculated from returns. Henceforth VaR calculations may refer to either of the two.

in the recent literature as a risk measure by Kou et al. (2013) and Kou and Peng (2014). Kou et al. (2013) and Kou and Peng (2014) defined the median shortfall as the median of the α tail distribution of Y. If Y has a cumulative distribution function (cdf) F then

$$F_\alpha(u) := \begin{cases} 0, & \text{for } u < VaR_\alpha; \\ \frac{F(u)-\alpha}{1-\alpha}, & \text{for } u \geq VaR_\alpha. \end{cases}$$

is the α-tail distribution then $MS_\alpha(Y) = F_\alpha^{-1}(1/2)$ is the median shortfall. When working with loss and profit distributions the risk measures are determined in the left tail and then it has been proved that $MS_\alpha(Y) = VaR_{\frac{\alpha}{2}}(Y)$ for any critical level α.

2.2 A Critical Comparison of VaR, MS and ES

An axiomatic approach[7] to derive risk measures under Basel 2 and Basel 3 is described in Kou et al. (2013). There is an extensive literature comparing the theoretical advantages and disadvantages of VaR and ES. The discussion is centered on whether a general risk measure ρ satisfies the well-known coherence conditions (Acerbi, 2004; McNeil et al., 2015) over the space of losses and profits (or equivalently returns) $\Pi = \{Y : E(Y^2) < \infty\}$: monotonicity, that is for two positions with values such that $Y_1 \geq Y_2$ the risks are also ordered $\rho(Y_1) \leq \rho(Y_2)$; the sub-additivity or merger risk reduction condition saying that $\rho(Y_1 + Y_2) \leq \rho(Y_1) + \rho(Y_2)$; homogeneity or scaling of risk condition$\rho(\lambda Y) = \lambda\rho(Y)$ for any $\lambda \geq 0$; and the risk-free risk reduction $\rho(Y + a) = \rho(Y) - a$ for any $a \in \mathbb{R}$ that requires adding cash to reduce risk.

Apart from monotonicity all the other properties can be called into question[8] from

[7] Farkas et al. (2014) analyse capital requirements for bounded financial positions based on Value-at-Risk and Tail-Value-at-Risk acceptability and show that a theory of capital requirements allowing for general eligible assets is richer than the standard theory of cash-additive risk measures. Moreover, they highlight that general capital requirements display a wider range of behaviors in terms of finiteness and continuity than classical cash-additive risk measures.

[8] It can be argued that the four conditions above reflect more risk preferences and as such the risk measures should be understood in relation to various stochastic orders. In essence that means using as a partial order among payoffs some stochastic order "$\preceq$" such that the monotonicity condition becomes $Y_1 \preceq Y_2$ implies $\rho(Y_1) \geq \rho(Y_2)$, and then the risk measure ρ is called coherent with respect to the stochastic order $\preceq$. Rémillard (2013) highlighted that VaR is coherent w.r.t simple stochastic order while ES is coherent w.r.t hazard rate order.Foster and Hart (2009) define a measure of riskiness

a pure financial point of view. The sub-additivity condition has been introduced on the basis that a merger cannot increase risk, see Artzner et al. (1999). However, this may not be the case as the following heuristic example suggests. Assume that a company A is taken over by a company B. Company A has 100 million dollars liabilities serviced at LIBOR plus 1% based on their credit rating AA while company B has 20 million dollars liabilities serviced at LIBOR plus 5%, again based on their rating BB. After the merger the company B will have liabilities 120 million but since they are not going to change their business model, their rating is roughly the same, or maybe only slightly upgraded to BBB so when they will have to pay 100 million dollars inherited from company A they will rollover their debt at LIBOR plus 4%. From a risk perspective, their cost of financing has been improved by 1% on their own 20 million but has deteriorated by 3% on 100 million dollars. This post merger situation will improve only over time but not immediately and hence, the merger leads to a company that is in a worse position than before. Garfinkel and Hankins (2011) showed that cash flow uncertainties lead firms to integrate vertically so our heuristic example is representative for risk management in mergers.

The homogeneity condition says that leverage increases risk only linearly. The subprime crisis has taught us that this is not the case. The risk-free reduction implies that adding cash to a risky portfolio must have the effect of reducing risk of that portfolio by the same amount. This seems correct in a world where risk-free rates are positive[9] but this view has been contradicted recently by the negative nominal rates in a series of countries such as EU, Japan and Switzerland.

The claimed superiority of ES over VaR was based on the verification of sub-additivity property. However, it is worth mentioning that Garcia et al. (2007), Ibragimov and Walden (2007) and Ibragimov (2009) demonstrate that actually VaR is subadditive under the family of infinite variance stable distributions when the mean

of an asset that is detached from the decision maker.

[9]The proper condition seems to be defined still in Artzner et al. (1999) where due consideration to the risk-free rate r is taken.

return is finite. Danielsson et al. (2013) generalized those results and found sufficient conditions for VaR to be subadditive in the relevant tail region when asset returns have a multivariate regular variation, for both independent and cross sectionally dependent returns provided the mean is finite. They propose an extreme value theory-based estimator which corrects for most empirical subadditivity failures because the results of extreme value theory ensure that, regardless of the underlying distribution, when the distribution of returns is fat tailed, the asymptotic tail follows a power law. Hence, this approach will produce estimated VaR value that are more robust to the uncertainty associated to the target quantile, and therefore avoiding greatly subadditivity violations. Dhaene and Salahnejhad (2015) remarks that ensuring subadditivity in the tail region is conditioned on working sufficiently deep into the tail, in order to be able to apply the Fellers convolution theorem. However, from a practical point of view being deep inside the tail leads to a major decrease of the number of observations which will increase estimation uncertainty.[10] Dhaene et al. (2006) proved that coherent risk measures can be too subadditive, that is they may imply an increase of the shortfall risk in case of a merger.

Danielsson and Zhou (2015) reviewed the accuracy of risk forecasts as measured by VaR and ES and they concluded that VaR is superior. They showed that for longer horizons a half century of data is needed for standard estimators to satisfy the asymptotic conditions, thus questioning the reliability of methodology promoted by the BCBS. Cont et al. (2010) focused on computational stability to perturbations of the model and concluded that the historical ES exhibits unavoidable instabilities[11] that do not arise in the case of historical VaR, questioning whether ES is superior ultimately to VaR. Ahn et al. (1999) described how an institution aiming to minimize its VaR can achieve an optimal risk control using options, the cost/VaR frontier being

[10] Dhaene and Salahnejhad (2015) investigated the sub-additivity of VaR and solvency capital requirements for insurance line business and they observed that the uncertainty of VaR estimation is not always monotonically increasing through the tail and it may change monotony as it goes deeper into the tail.

[11] ES is much more sensitive to adding an extra data point than VaR, and it is also sensitive to the size of the extra data observation.

linear. Kondor and Varga-Haszonits (2010) proved that if there is a pair of portfolios such that one of them dominates the other in a given sample–and this occurs with finite probability even for large samples–, then it is impossible to determine an optimal portfolio under ES and furthermore the risk measure diverges to $-\infty$.

Another argument frequently invoked in favor of ES is that VaR ignores the magnitude of the losses greater in absolute value than VaR, so naked speculative positions are not captured by VaR and hence allows the traders to take extreme risks. However, if the models used for risk management are poor in estimating tail risk then it is better to use VaR rather than an ES measure contaminated with estimation risk. Furthermore, in order to estimate reliably ES at the same level with VaR, a much larger sample size is needed and moreover, the estimation of ES is computationally more costly for fat-tailed distributions (Yamai and Yoshiba, 2005). Christoffersen and Goncalves (2005) also confirmed that ES measures are generally less accurate than VaR measures and the confidence bands around ES are also less reliable.

On the practical side the quality of a risk measure is qualified based on backtesting against realised losses. Recently it has been proved (Gneiting, 2011; Ziegel, 2014) that ES is not an elicitable measure of risk. It has been argued that this is equivalent to the impossibility of backtesting ES. Since VaR is elicitable and hence backtestable, the comparison with ES seems to have reversed. However, the lack of elicitability is not proof that backtesting ES is impossible and McNeil et al. (2015), Du and Escanciano (2016) describe feasible methods to backtest ES.

Davis (2014) showed that VaR is a consistent risk measure and it has theoretical properties that ES does not have. In a similar vein to Diebold-Mariano tests, a risk measure is categorised as performing well if a criterion depending only on realized values of data and the numerical values of predictions, is satisfied in line with the weak prequential principle of Dawid (1984) referring to probability forecasting. Davis proves that the consistency of quantile forecasting is obtained under essentially no conditions on the mechanism generating the data. Kou et al. (2013) argued also that MS is more robust than ES in terms of robust statistics tools such as influence functions, asymptotic breakdown points and finite sample breakdown points and So and Wong (2012) pointed out that MS is coherent when the losses or returns have elliptical contoured probability distributions.

Some less-known properties that risk measures should have in general[12] have been described in Albanese and Lawi (2004). Here we select two of these properties. The first one is called the *irrelevance of positive gains* and it is defined by requiring for a random payoff with loss or profit X having zero mean that $\rho(Y) = \rho(\min(Y, 0))$, so the risk is only represented by the negative return. It can be shown, see the Online Appendix, that the cumulative distribution function of the negative part $Z = \min(Y, 0)$ of a random payoff Y has the following cumulative distribution function

$$F_Z(z) = \begin{cases} 1, & \text{if } z \geq 0; \\ F_Y(z), & \text{if } z < 0. \end{cases} \tag{1}$$

where F_Y is the cumulative distribution function of Y. For a given critical level α such as $\alpha = 5\%$ or $\alpha = 1\%$, *in general*[13], if $F_Y(0) > \alpha$, it follows that the α-quantile for Y is equal to the α-quantile for Z, because the corresponding cdf for Y and Z are identical on the negative semiaxis of real numbers. This implies that $VaR_\alpha(Y) = VaR_\alpha(\min(Y, 0))$ and since this identity is then true for any critical level $u < \alpha$ it is also true that $MS_\alpha(Y) = MS_\alpha(\min(Y, 0))$, and using the known formula linking ES to VaR, see(8) in the next section, it also follows that $ES_\alpha(Y) = ES_\alpha(\min(Y, 0))$. However, if $F_Y(0) < \alpha$ then the α-quantile of Y is a positive number and furthermore the α-quantile of $\min(Y, 0)$ does not exist therefore invalidating the condition of positive gains for both VaR and ES. Nevertheless if it also happens that $F_Y(0) < \alpha/2$, then $MS_\alpha(Y)$ does exist and it is also equal to $MS_\alpha(\min(Y, 0))$, so the irrelevance of positive gains may still hold for MS[14].

[12] More interesting theoretical properties are discussed in Foster and Hart (2009).

[13] Our condition is more relaxed than the condition of returns centered at zero required by Albanese and Lawi (2004)

[14] If $F_Y(0) > \alpha/2$ then the same technical problems presented for VaR also applies for MS.

The second property is the relative diversification of risk is captured by the *monotonicity of specific risk* and it says that if a position Y_1 is cloned into independent copies $Y_1, Y_2, \ldots, Y_n, \ldots$ then for any integers $0 < m \leq n$ we have that

$$\frac{1}{n}\rho(Y_1 + Y_2 + \ldots Y_n) \leq \frac{1}{m}\rho(Y_1 + Y_2 + \ldots Y_m) \tag{2}$$

so more positions of the same kind should reduce the risk per unit of trade. Here we can prove the following result.

Proposition 2.1. *If m and n are large enough, for any $\alpha \in (0, 0.5)$, VaR_α and ES_α are both monotonic to specific risk.*

A detailed proof is given in the Online Appendix. The benefit of reducing risk per unit of trade may also occur when the distribution of payoffs is not related to the normal distribution. For example, if $Y \sim Cauchy(\mu, \gamma)$, one can prove–see the Online Appendix–, that the condition (24) is satisfied for both VaR and ES, with equality.

Proposition 2.2. *The risk per unit of trade does not always decrease with the increase of number of trades of the same kind.*

In Table 1 we collected the properties of the three main risk measures currently being advocated in the literature by various schools of thought. Overall, we may conclude that MS is at least as good a risk measure as ES while it is also more flexible computationally.

[Table 1 about here.]

We would like to raise here another very important point regarding the veridicity of a particular algebraic condition for a risk measure. Without loss of generality we shall consider ES as an example of a risk measure and subadditivity as an example of a required condition. It is known Acerbi and Tasche (2002); McNeil et al. (2015) that ES verifies the subadditivity condition $ES_\alpha(Y + Z) \leq ES_\alpha(Y) + ES_\alpha(Z)$ for any positions Y and Z. If we denote by $\widetilde{ES}_\alpha(Y)$ the estimator of $ES_\alpha(Y)$ calculated from a sample $\{Y_1, \ldots, Y_n\}$ then relative to given samples of data

$$\widetilde{ES}_\alpha(Y) = ES_\alpha(Y) + \varepsilon_Y \tag{3}$$

$$\widetilde{ES}_\alpha(Z) = ES_\alpha(Z) + \varepsilon_Z \tag{4}$$

$$\widetilde{ES}_\alpha(Y+Z) = ES_\alpha(Y+Z) + \varepsilon_{Y+Z} \tag{5}$$

It is possible to *reverse* the in-sample version of the in population condition, that is

$$\widetilde{ES}_\alpha(Y+Z) > \widetilde{ES}_\alpha(Y) + \widetilde{ES}_\alpha(Z) \tag{6}$$

if the estimation errors from the respective samples come out such that

$$\varepsilon_{Y+Z} - \varepsilon_Y - \varepsilon_Z > ES_\alpha(Y) + ES_\alpha(Z) - ES_\alpha(Y+Z) \tag{7}$$

Since for various estimators of risk measures it is impossible to control the estimation errors, for all practical purposes, it is impossible to guarantee that a required condition is also satisfied across all samples. This also goes in reverse. A condition may fail at the population level equivalent to assuming that there is no sampling error, but the sample estimates may actually verify the condition, once again because of the sampling errors ε. This is true for ES, for VaR, and for any other risk measure.

3 Model Validation of Risk Calculations

In this paper we shall assume that X_t represents the value at time t of an asset, such as an equity index or a foreign exchange, and that the value of the asset at a fixed horizon h can be described by a random variable $X_{t+h} = X_t e^{Y_{t+h}}$, where Y_{t+h} denotes the log-returns of the asset on the interval $[t, t+h]$. For risk management purposes we are interested in the left tail of the distribution of the direct losses and profits

$X_{t+h} - X_t$. For calculation purposes the central place is taken by the the distribution of log-returns Y_{t+h}, under various models.

Focusing the attention on the returns Y the VaR at some critical level α can be conceptualised[15] by the α-quantile of the distribution of returns F, with negative sign. If the quantile q_α is such that $q_\alpha = -VaR_\alpha$ then the ES at the same critical level is given by

$$ES_\alpha = -\frac{1}{\alpha} \int_0^\alpha F^{-1}(u)du \tag{8}$$

Acerbi and Tasche (2002) proved[16] that $ES_\alpha = \lim_{n\to\infty} -\frac{1}{\lfloor n\alpha \rfloor} \sum_{i=1}^{\lfloor n\alpha \rfloor} Y_{[i:n]}$, where $Y_{[i:n]}$ represents the i-th order statistics in a sample of size n and $\lfloor a \rfloor$ denotes the smallest integer not greater than a. Hence, a useful estimator[17] of ES is given by

$$\widehat{ES}_\alpha = -\frac{1}{\lfloor n\alpha \rfloor} \sum_{i=1}^{\lfloor n\alpha \rfloor} Y_{[i:n]} \tag{9}$$

3.1 Quantile Risk Measures and Distribution-Free Confidence Intervals for VaR

In this paper we consider parametric models as well as a nonparametric approach for estimating risk measures. For the former, we use the gaussian model, still widely applied in the finance sector, as well a model better equipped to deal with fat tails such as the NIG, which is representative for the Lévy models applied in finance (Andersson, 2001; Venter and de Jongh, 2002; Aas et al., 2006; Ghysels and Wang, 2014). Moreover, given the well-known conditional heteroskedastic patterns in financial re-

[15]We can assume without loss of generality that the cumulative distribution function F for the log-returns is absolutely continuous with a probability density f.

[16]For a more precise calculation from a sample of returns data $\{Y_1, \ldots, Y_n\}$, because the empirical distribution function generated $F_n(u) = \frac{1}{n}\sum_{i=1}^n 1_{\{u \geq Y_i\}}$ is not invertible, Inui and Kijima (2005) suggested to employ the lower empirical distribution value given by the piece-wise constant function equal to $Y_{[k:n]}$ order statistic when $\frac{k-1}{n} < \alpha \leq \frac{k}{n}$. This gives a direct estimator for VaR_α as $-Y_{[k:n]}$ but this estimator is known to carry a positive bias for small critical levels α.

[17] Danielsson and Zhou (2015) point out that quantile estimators are biased in small samples. In our paper all samples are large enough to avoid this bias and we prefer to use quantile estimators for which we can construct confidence intervals and therefore be able to quantify the uncertainty of parameter estimation.

turns, we also take into account additional models such as the GARCH(1,1) and the GJR-GARCH(1,1) with normal innovations. Many banks seem to prefer model free approaches[18] since it is unlikely that a family of models will perform well across all asset classes and will be able to cover new emerging asset classes.

In order to derive nonparametric market risk measures for an investment let us assume that the log-returns of the asset are characterised by an arbitrary continuous and strictly increasing cumulative distribution function $Y_{t+h}|\mathcal{F}_t \sim F$. Then, the daily VaR and the daily ES at horizon h and critical level α can be computed as follows

$$VaR_t^{nonp}(X_{t+h} - X_t) = X_t - X_t e^{F^{-1}(\alpha)} \tag{10}$$

$$ES_t^{nonp}(X_{t+h} - X_t) = X_t - \frac{1}{\alpha} X_t \int_{-\infty}^{F^{-1}(\alpha)} e^Y dF(Y). \tag{11}$$

VaR and ES can be conceptualised as functions of the α-th quantile of the distribution of the asset returns, which is notoriously difficult to estimate. Nevertheless, having a sample of past returns of the asset available, it is possible to estimate $F^{-1}(\alpha)$ relying only on the sample quantile $y_{[\lfloor n\alpha \rfloor :n]}$, i.e. a nonparametric approximation. Thus, VaR and ES can be estimated as follows:[19]

$$\widehat{VaR}_t^{nonp}(X_{t+h} - X_t) = X_t - X_t e^{\hat{F}^{-1}(\alpha)} = X_t - X_t e^{y_{[\lfloor n\alpha \rfloor :n]}} \tag{12}$$

$$\widehat{ES}_t^{nonp}(X_{t+h} - X_t) = X_t - \left[\frac{1}{\lfloor n\alpha \rfloor} X_t \sum_{i=1}^{n} e^{y_i} 1_{\left\{ y_i \in \left[y_{[1:n]}, y_{[\lfloor n\alpha \rfloor :n]} \right] \right\}} \right]. \tag{13}$$

VaR measures estimated according to formula (12) are subject to sampling estimation error. One possibility in dealing with this issue is to construct, for example, a 95% confidence interval for VaR[20]. Assuming that the true risk measure lies inside the confidence interval, the only inference one can make is that it can be any point of the interval. The order statistics calculus allows the construction of a *distribution-*

[18]O'Brien and Szerszen (2014) describe a sample of U.S. banks for which VaR and ES are calculated. From this sample 60% employ the nonparametric or historical simulation method.

[19]Chen (2008) proposed $\widehat{ES}_\alpha = -\overline{Y}_{[\kappa:n]}$ as a valid estimator of ES, with $k = n\alpha + 1$.

[20]Gupta and Liang (2005) estimated confidence intervals for VaR using profile likelihood methods.

free confidence interval for quantiles, and hence for VaR measures. This approach has been pioneered in the risk management literature by Dowd (2006, 2010) and it is expanded on here. An important result from order statistics (David and Nagaraja, 2003) is that for any $1 \leq i_1 < i_2 \leq n$

$$Prob\left(Y_{[i_1:n]} \leq q_\alpha \leq Y_{[i_2:n]}\right) = \sum_{j=i_1}^{i_2-1} \binom{n}{j} \alpha^j (1-\alpha)^{n-j}. \tag{14}$$

While this result can be used to determine directly a distribution-free confidence interval for VaR, there may exist several combinations of order statistics $Y_{[i_1:n]}, Y_{[i_2:n]}$ that make the probability in (14) equal to the desired confidence level. One can apply the approach by Hutson (1999) that allows to identify the optimal sample size in order to get significant confidence intervals and to select the optimal[21] pairs (i_1, i_2) once n and α have been fixed.

Here, we propose to construct distribution-free confidence intervals at $1-\beta$ level for VaR of an investment in an asset using $y_{i_1} = Y_{[i_1:n]}$ and $y_{i_2} = Y_{[i_2:n]}$ satisfying $Prob\,(y_{i_1} \leq q_\alpha \leq y_{i_2}) = 1-\beta$

$$CI_{1-\beta}^{VaR_t^{nonp}}(X_{t+h} - X_t) = (X_t - X_t e^{y_{i_1}}, X_t - X_t e^{y_{i_2}}) \tag{15}$$

3.2 Parametric Confidence Intervals for VaR

The order statistics calculus provides additional tools when the returns of the asset are assumed to be distributed according to a certain parametric law F.

If $F_{[i]}(u) = P(Y_{[i:n]} \leq u)$ is the cumulative distribution function of the i-th order statistic then $F_{[1]}(u) = 1 - [1 - F(u)]^n$ and $F_{[n]}(u) = [F(u)]^n$. The distribution of the order statistics of any order j can be derived (David and Nagaraja, 2003) as

$$F_{[j]}(u) = \mathcal{B}_{F(u)}(j, n-j+1) \tag{16}$$

[21] Optimal here means that the distance between i_2 and i_1 is minimal.

where $\mathcal{B}_U(a,b) = \frac{\int_0^U t^{a-1}(1-t)^{b-1}dt}{B(a,b)}$ is the incomplete beta function and $B(a,b)$ is the beta function. Thus, the probability density function of the j-th order statistics is

$$f_{[j]}(u) = \frac{1}{B(j, n-j+1)} F^{j-1}(u)[1-F(u)]^{n-j} f(u) \tag{17}$$

where f is the corresponding density to F.

Once a parametric distribution model is specified for the returns of the asset, one can derive not only confidence intervals but also the entire distribution associated with the VaR measure. For example, if f is gaussian or NIG, we can simply replace their formulae in (17) and obtain the formula of the density of the corresponding VaR estimate. In this way the risk manager can determine how uncertainty around VaR is distributed. A confidence interval will assume that all values inside the interval are equally likely, whereas seeing the probability density function allows a better assessment of risk. On the other side, confidence intervals give a simple and straightforward assessment of model risk related to the estimation of VaR. For this reason we derive the expressions for the confidence intervals at $1-\beta$ level for VaR of an asset:

$$CI_{1-\beta}^{VaR_t^{nonp}}(X_{t+h} - X_t) = \begin{pmatrix} X_t - X_t e^{\mathcal{B}^{-1}_{F(\beta/2)}(\lfloor n\alpha \rfloor, n - \lfloor n\alpha \rfloor + 1)}, \\ X_t - X_t e^{\mathcal{B}^{-1}_{F(1-(\beta/2))}(\lfloor n\alpha \rfloor, n - \lfloor n\alpha \rfloor + 1)} \end{pmatrix}. \tag{18}$$

3.3 A Measure of Unexpected Losses

If VaR is estimated based on[22] the order statistics $Y_{[v:n]}$, there is a direct advantage to estimate the median shortfall (MS) from the truncated sample $Y_{[1:n]}, \ldots, Y_{[v-1:n]}$, which is already an ordered sample. Estimating both the expected losses and the unexpected losses as a quantile measure at different orders allows the calculation of the bivariate joint distribution of $(Y_{[\kappa_1:n]}, Y_{[\kappa_2:n]})$ where $\kappa_1 = n \times \alpha_1$ and $\kappa_2 = n \times \alpha_2$,

[22] When v is even then the estimate of the median shortfall is $Y_{[v/2:n]}$ and when v is odd then the sample estimate is $\frac{1}{2}\left[Y_{[(v-1)/2:n]} + Y_{[(v+1)/2:n]}\right]$. A simpler estimator is to always consider $Y_{[\lfloor v/2 \rfloor:n]}$ as the median shortfall estimate and henceforth we will use this simpler estimator in order to treat the MS as a quantile.

respectively. This distribution is

$$F_{[\kappa_1,\kappa_2]}(u,v) = \sum_{k=\kappa_2}^{n} \sum_{s=\kappa_1}^{\kappa} \frac{n!}{s!(k-s)!(n-k)!}[F(u)]^s[F(v)-F(u)]^{k-s}[1-F(v)]^{n-k} \quad (19)$$

for any $u < v$. This implies that any two quantile order statistics estimators at different critical levels *are not independent*. Hence, in risk management terms, it is better to look at VaR and MS as a pair of tools for gauging risk.

For capital requirements purposes we consider using the *excess difference* between the MS estimate and the VaR estimate. This measure will account for the reserves that a bank will need to carry to safeguard from unexpected losses. We believe that VaR is indicative for market risk under normal market conditions and this "expected" risk should be mitigated on a day to day basis by adjusting the positions on the bank's balance sheet. It is a watermark system that is negotiated daily by repositioning on the market. For the unexpected losses, for example being long on S&P 500 in 2008, only capital reserves in cash or very liquid assets could have mitigated against those sudden losses caused by the equity crash.

The MS sample estimate being an order statistic, denoted henceforth by $Y_{[m]}$, with $m < v$, drives the capital requirements reserves that should be deposited. The actual capital requirements measure is determined by the negative of the difference

$$D = \left(Y_{[v]} - Y_{[m]}\right) \quad (20)$$

accounting for unexpected losses. Our approach is reminiscent of the ΔCoVaR approach of Adrian and Brunnermeier (2016) for quantifying systemic risk and of Colletaz et al. (2013) who validate risk models looking at stressed values of VaR. The measure defined in (20) uses a different critical level from the quantile order used in the ΔCoVaR which takes $v = 0.5$ whereas we use $\alpha \ll 0.5$, and it is also by definition not the same as the risk map measure described in Colletaz et al. (2013) where m takes a series of values in order to create a vector of stressed VaRs, whereas our m

comes out by calculation to be precisely equal to $v/2$.

If $Y_{[m]} = z$ and $Y_{[v]} = y$ then $D = y - z$ then using the order statistics calculus the distribution of D is given by the density

$$q(d) = K \int_{-\infty}^{\infty} F^{m-1}(z) \left[F(z+d) - F(z)\right]^{v-m-1} \left[1 - F(z+d)\right]^{n-v} f(z) f(z+d) dz \tag{21}$$

where $K = \frac{n!}{(m-1)!(v-m-1)!(n-v)!}$ is just a normalising constant factor. The probability density presented in equation (21) can be used to construct the p-value for the measure D of unexpected losses. For a given model represented by the cdf F, the p-value of observing a given value $D = d$ is equal to

$$P_F(D \leq d) = \int_0^d q(u) du \tag{22}$$

If this p-value shows that the observed data is in the extreme tails of the density given in (21) then the risk manager or the regulator can reject the model used to capture unexpected losses. This tool can be very useful to monitor model performance and hence our methodology can also be applied to risk measures calculations for a single asset or a portfolio, across business units or for a bank, insurance company, hedge fund and it can also be applied to credit risk calculations, in a similar vein to Wilkens and Predescu (2015), where Value-at-Risk-type measures are sought for losses over a one-year capital horizon at an extreme tail (99.9%).

4 Data Description

In this section we describe the data used in this paper. For our empirical application we consider the daily time-series of S&P500 and the USD/GBP exchange rate over a thirty year time period 02/01/1984 – 10/07/2014. For the USD and GBP risk-free interest rates we used the middle rate on 3-months deposits determined on the Eurocurrency market.

The analysis focuses on three risk measures: VaR, ES and MS at 1%, 2.5% and 5% level, along with the 95% confidence intervals for VaR and MS obtained according to the theoretical approaches explained in the previous section. The data for the empirical analysis have been retrieved from Thomson Reuters Datastream.

We develop our analysis by using a daily rolling window approach with a window of 520 days (roughly two years) for the estimation of the relevant parameters. The choice of a rolling window of 520 days is not casual; beside the fact that the Basel Accord suggests to use a sample size of two years in VaR estimation, we actually checked the appropriateness of such window size following the approach of Hutson (1999). In Figure 1 we plot the quantiles used in order to derive the 95% distribution-free confidence intervals for VaR and MS and it appears that after 520 days the model-free confidence intervals begin to stabilize. Hence, roughly two years of daily data seems to be the minimum sample size a risk manager should use in order to be able to asses the uncertainty of risk estimates[23].

[Figure 1 about here.]

We derive risk measures by modeling the log-returns of the S&P 500 and of the USD/GBP exchange rate according to five different models: the nonparametric i.i.d. model, the gaussian distribution model, the NIG distribution model, the GARCH(1,1) model and the GJR-GARCH(1,1) model with gaussian innovations. Ornthanalai (2014) pointed out Lévy models may be superior to standard gaussian models as asset pricing models. Here we explore whether the same is true from a risk management perspective. The usefulness of this approach is gauged by performing a battery of backtests for the estimated risk measures and their extensions counterparts that include model risk. Subsequently we discuss how model risk can be taken into account in the computation of capital requirements.

[23]Greater accuracy can be achieved at the expense of increasing the sample size. This may not be possible for the majority of assets traded on financial markets so we preferred using a sample size that can be applied more widely in practice.

5 Selecting Risk Measures and Models

5.1 The Order Statistics Approach

In Figures 2–3 we show, for a long position on the S&P 500 and a long position on the USD/GBP exchange rate, the time series of the nonparametric VaR and the nonparametric MS at 1% and 5%, along with their 95% distribution-free confidence intervals. For both assets the higher the critical level at which the two risk measures are computed the narrower is the confidence interval, as more information regarding the left tail of the asset returns distribution is exploited. Our newly proposed distribution-free confidence interval based on quantiles is able to capture very well the non-parametric point estimate of all three risk measures, both in normal times and in turbulent times. Our analysis confirms the conclusion of Danielsson et al. (2016) that during calm periods various risk models produce close forecasts, therefore rendering model risk as small, while during market distress model risk increases significantly.

[Figure 2 about here.]

[Figure 3 about here.]

In Figure 4 we plot the time series of the nonparametric ES and the nonparametric MS, the two competing tail risk measures, at 1%, 2.5% and 5%. We notice that, for all assets investigated, the MS is generally smaller than the ES in normal times, but during crises MS may exceed the risk levels indicated by ES, particularly at the 1% critical level. This is clearly true for the crisis that followed the subprime debacle of 2007 for equity and FX asset classes. The equity crisis of Black Monday 1987 is rather interesting, with the MS indicating a much lower level of risk than the ES risk measure. A possible explanation of the sources[24] of this event indicate that this particular case was exogenous to the equity market.

[24] In August 1987 the Dow Jones index was trading at 2722 points which was 44% over the previous year's closing of 1895 points. On October 14, the Dow Jones index dropped 95.46 points to 2412.70, and another 58 points the next day, down over 12% from the August 25 all-time high. During this period the United States was engaged in a war with Iran and on Thursday October 15, 1987, Iran hit an American-owned supertanker with a missile. The next morning, Iran hit another

[Figure 4 about here.]

On the other hand, on the currency market the Black Wednesday event of 16th September 1992, when the British government withdrew the pound sterling from the European Exchange Rate Mechanism, was an endogenous[25] market risk event. The graph in Figure 4 reveals that, for the USD/GBP exchange rate, prior to this event the MS at 1% was consistently above the ES at 1% for a significant period of time, suggesting perhaps that an extreme event was likely. After this event occurred the MS is equal to ES at all three confidence levels. This evidence supports our choice of using the MS as proper measure of tails risk.

The graphs in Figure 4 suggest that MS is a better measure of risk than ES since it is less conservative in good times and it can signal higher levels of risk before crises. However, nonparametric calculations for risk measures may not be feasible for all asset classes since nonparametric estimation requires large samples of data. The question with using parametric models is which model is appropriate. Should banks and financial institutions rely on the gaussian distribution given the high level of applicability and small number of parameters to estimate or should they consider some of the more recent distributions/processes advocated in the markets such as Lévy processes? Or is it better to bring into play the GARCH family of models?[26] In this paper we shall compare these approaches and conduct later on backtesting in order to identify the correct procedure for the assets investigated.

American ship with another missile. On Friday 16 October, all the financial markets in London were closed due to the Great Storm of 1987. The Black-Monday crash began unfolding like a tsunami from the Far Eastern markets on the morning of Monday October 19. Hence, the first to take action were traders in Asia and only afterwards reaching London and European countries on its way to the U.S. Over this time two U.S. missiles bombarded an Iranian oil platform in retaliation to Iran's previous missile attacks. The programme trading invoked in the risk management literature most likely created the exposure for the event to impact, it was likely not the cause. This is a perfect example where stock market risk (programme trading) was bundled with political risk (the US-Iran war) and catastrophic risk (insurance losses caused by the Great Storm were estimated at $2 billion).

[25] The event that the UK government suspended Britain's membership of the European Exchange Rate Mechanism was endogenous. On the other hand the fact that the Chancellor Norman Lamont raised interest rates from 10% to 12%, then to 15%, and authorised the spending of billions of pounds to prop up the sterling could be also seen as an exogenous event to the FX markets.

[26]Note that since GARCH models are not i.i.d it is not possible to apply the approach presented in this paper based on the order statistics and the analyst must then rely solely on the backtesting exercise in order to validate the model.

We illustrate in Figure 5 and Figure 6 the historical evolution of market risk as represented by VaR and the tail risk as represented by MS, under calculations based on the gaussian distribution and the NIG model, at 1% critical level [27]. The confidence interval based on order statistics that we advocate in this paper can be used as a model screening tool in risk management. The results presented in Figure 5 show that, under the gaussian model, the non-parametric measures of risk lie outside the confidence bands, particularly in the stress periods. This does not happen when similar calculations are performed under the NIG distribution (Figure 6), suggesting that this particular case of the Lévy distribution is much better suited to risk measuring calculations. Furthermore, as we can see from the results depicted in Figure 5 the level of risk in comparative terms seems to be underestimated by the gaussian distribution.

[Figure 5 about here.]

[Figure 6 about here.]

One great advantage of being able to calculate confidence intervals for the point estimates of the risk measures is that we can use the intervals boundaries to introduce a measure of model parameter estimation risk and to adjust easily risk calculations for this type of model risk. Thus, following our approach parameter estimation risk can be easily accounted for and risk measures can be easily adjusted for this additional risk, as requested by regulators.

5.2 Model Validation For Unexpected Losses

When the distribution of the returns is available analytically we can derive the density function and the cumulative distribution function of the difference between the VaR and the MS order statistics estimators. The algebraic details are provided in

[27] A wider range of empirical results including similar calculations for risk measures estimated at 5% critical level are available from the authors upon request. These results are analogous to those obtained for risk measures estimated at 1% critical level.

the Appendix. This tool allows us to compute the p-values under the null hypothesis that the observed differences between the VaR and the MS estimates belong to the conditional distribution. In this way it is possible to assess whether the model envisaged for the distribution of the asset returns is appropriate or not for capturing unexpected losses. To this end we compute the p-values as series of probabilities $P(D \leq d)$, where d represents the realized difference representing unexpected losses, and we validate the model as being correct for measuring unexpected losses, at the critical level 5%, if these p-values are consistent between 2.5% and 97.5%, and reject the model when these p-values are either below 2.5% and larger than 97.5%.

[Figure 7 about here.]

The graphs in Figures 7 show[28] the p-values obtained for the unexpected losses measure as calculated with the gaussian distribution and the NIG distribution, respectively. We can infer unequivocally that the NIG distribution is superior in terms of capturing unexpected losses using the VaR and MS as pointwise risk measures. The gaussian distribution has far too many observed losses in the extreme of the distribution of the measure of unexpected losses D indicating a disconnect between the way the gaussian model forecasts both expected and unexpected losses.

The tool described in this section is the first one in the literature, to our knowledge, that facilitates model validation looking at both expected and unexpected losses. Furthermore, given that various methodologies for measuring systemic risk[29] is based essentially on the difference between the median, which is the VaR at $\alpha = 50\%$ and a stressed VaR at $\alpha = 1\%$, it seems feasible to adapt our tool for model validation in the context of systemic risk measurement as well. Since in Colletaz et al. (2013) a standard VaR and a stressed VaR at very low alpha such as 0.1% are employed to control the quality of risk measures, it seems feasible to adapt our tool for model

[28]Similar graphs for the 5% VaR and MS, depicting a similar outcome that the NIG is a superior risk management model, are available from the authors upon request.

[29]Excellent reviews of these methodologies are contained in Danielsson et al. (2016) and Bisias et al. (2012).

validation to this framework. The model validation exercise may be applied to market, credit, operational, or systemic risk estimates, and even to assess the performance of the margin system of a clearing house.

6 Backtesting

Backtesting is very important in risk management, particularly for the regulator. We are going to apply a battery of backtesting procedures –including the FOEL test, Kupiec's POF Test, Christoffersen's Independence Test and Christoffersen's Conditional Coverage Test– to identify the method that is most consistent with the significance level. The Christoffersen's Independence test proposed by Christoffersen (1998) tries to handle the clustering of violations relying on a simple first-order Markov sequence. Christoffersen and Pelletier (2004) suggested a more sophisticated approach that takes into account the duration of time between consecutive violations. Later Berkowitz et al. (2011) extended this methodology to define a new Conditional Coverage Test. Engle and Manganelli (2004) proposed the dynamic quantile Conditional Coverage test[30] that allows instead to check high-order dependence. The backtests that have been described so far permit to evaluate the performance of a model for the calculation of VaR at a certain critical level (e.g. 1%). Pérignon and Smith (2010) proposed a multivariate generalization of the Kupiec's POF Test that allows instead to assess the accuracy of a model at any critical level. According to this multivariate unconditional coverage test, a vector of VaRs having same horizon but different coverage probabilities can be backtested jointly.

The backtesting results presented in Tables 2-3 indicate that for S&P500 the best methods are the VaR at 5% calculated with the GJR-GARCH-N distribution and the MS at 1% calculated with the NIG distribution.[31] For the USD/GBP rate, from

[30] To implement the test we follow Engle and Manganelli (2004), Dumitrescu et al. (2012) and Subba Rao et al. (2012)

[31] If the regulator insists on a specific critical level like 1% and a specific measure like VaR then the best methods are the historical and the NIG, adjusted for model risk.

Tables 4-5 it can be concluded that VaR at 1% computed with GARCH(1,1) model and the MS at 1% calculated with the NIG distribution performed the best.[32] Hence, both VaR and MS can be reliable measures of market risk, at different confidence levels, and the NIG model seems to be a reliable model. Adjusting for model risk leads to more conservative measurements but our backtests indicate that overall this may be more desirable than working with GARCH models. Investors may prefer GARCH models as they may be less risk averse than a regulator and hence they are happy to work with VaR at 5%. On the other hand the regulator is more conservative by definition and she may prefer to work with MS at 1%. The rejection of the gaussian model is hardly surprising, confirming recent results using desk-level data, see Berkowitz et al. (2011). The results of the Pérignon and Smith Test in Table 6 suggest that the NIG distribution is an accurate model for the estimation of VaR and MS for both assets, while the nonparametric model works well for VaR and MS related to the USD/GBP exchange rate. This set of results emphasize that taking into consideration the uncertainty surrounding the risk measure calculations is equivalent to more conservative market risk estimation. While it may be tempting to suggest that all one needs to do is to use VaR at a more stringent critical level, the results need to be validated by backtesting.

[Table 2 about here.]

[Table 3 about here.]

[Table 4 about here.]

[Table 5 about here.]

[Table 6 about here.]

[32]In each table we compared the performance on different rows and retained those model/risk measure/critical level that had passed the largest number of backtests and with the most stars.

7 Capital Requirements

In the aftermath of the unprecedented cascade of financial crises over the period 2007-2011–subprime, liquidity, Libor, sovereign– new regulations have been developed to improve capital requirement calculations BCBS (2010, 2011). The role of capital requirements is to disincentivize bank shareholders from buying risky assets in order to capitalise on underpriced government bailout guarantees (Rochet, 1992). It is important to be able to gauge the appropriate level of capital requirements since undercapitalization harms profitable growth opportunities and the capitalization of a bank will be ultimately driven by the net impact of capital levels on the default option and the franchise value, as demonstrated by Barone-Adesi et al. (2014). We claim that capital requirements should be set in such way to cover expected losses as well as unexpected losses, caused by unforeseeable events, in order to ward off a possible default. At the same time, capital requirements should not be excessively conservative to allow banks to efficiently exploit the available resources.

7.1 Model Comparison of Capital Requirements Calculations

In what follows we develop a comparative analysis focusing on capital requirements obtained according to the currently in effect BCBS regulation, looking at expected losses and unexpected losses, and also considering the effect of adjustments for parameter risk estimation. Capital requirements should be considered (Cuoco and Liu, 2006) jointly with backtesting procedures to induce financial institutions to report the level of risk they take and to manage this risk by having adequate levels of capital. The reform ongoing regarding the capital reserves regulatory framework (Basel 2.5 and Basel 3) introduced some changes in the computation of market risk capital reserves. The banks must meet, on a daily basis, a capital requirement defined by

$$CR = max\left\{\widehat{VaR}_{t-1}, k \cdot \widehat{VaR}_{avg}\right\} + max\left\{\widehat{sVaR}_{t-1}, k_s \cdot \widehat{sVaR}_{avg}\right\} \tag{23}$$

where $\widehat{VaR}_{t-1}$ is the previous day VaR estimate, computed at 1% level and with an horizon of 10 working days (two weeks), $\widehat{VaR}_{avg}$ is the average of the daily VaR on each of the preceding sixty business days, computed at 1% level and with an horizon of 10 working days, k is a multiplication factor subject to an absolute minimum of 3, $\widehat{sVaR}_{t-1}$ is the previous day stressed VaR estimate, at the same level of confidence and same horizon. The stressed VaR has to be computed using historical data drawn from a continuous 12-months period of significant financial stress relevant to the bank's portfolio. Ignoring for a moment the stressed VaR, it is worthwhile to make a comparison between capital requirements defined on the basis of the first term of formula (23) and our measure of unexpected losses adjusted for parameter risk estimation. We are going to show that, even when ignoring stressed scenarios, the regulatory capital requirements are excessive.

Figure 8 show the two-week losses, the expected losses (computed according to the nonparametric VaR), the unexpected losses (calculated on the basis of the nonparametric MS), the unexpected losses adjusted for parameter risk estimation (i.e. the upper part of the 95% confidence interval for the nonparametric MS) and the capital requirements computed according to BCBS regulation ignoring stressed scenarios (first term of formula (23)) for an investment of \$100 in the S&P 500 and in the USD/GBP exchange rate. The unexpected losses adjusted for risk estimation are obtained as the upper part of the 95% confidence interval for the MS. More results are presented in the Appendix. The measure of unexpected losses adjusted for parameter risk estimation obtained under the nonparametric model are generally more conservative that those computed under the NIG assumption, especially in the two years following the 1987 stock market crash. The nonparametric method leads to a period of overhang in the aftermath of a market crash such as 1987, an undesirable characteristic since less immediate risk follows after a crash, as discussed in his papers by Daníelsson and his co-authors. This is not true in the case of the NIG model.

Recall that BCBS capital requirements as indicated in Figure 8 are calculated by discarding the second term of formula (23). Generally the stressed VaR is at least equal or greater than the current VaR. Because stressed VaR values are left out, the actual capital requirements computed according to the BCBS directives are at least double than those exposed the figures. The difference between the imposed Basel calculations and the values derived under a robust model such as NIG are not only economically and statistically significant, they are very large, persistently over time.

This analysis implies that the currently in effect BCBS regulation is *excessively conservative*, which is to an extent paradoxical since, as observed by Hart and Zingales (2011), governments may relax very stringent capital requirements when a bank is exposed to bankruptcy, in order to avoid political pressure. One possible explanation is that BCBS is actually not subordinated to any government and they want to be seen as being conservative towards risk. Baker and Wurgler (2015) emphasized that capital requirements do increase cost of capital, consequently creating a disadvantage for the regulated banks vis-a-vis the shadow banks. Furthermore, if banks are required to maintain more capital than necessary, the risk is shifted onto the shareholders, as pointed out by Admati et al. (2013); Admati and Hellwig (2013).

[Figure 8 about here.]

The capital requirements calculations under the NIG model reflect a more stable analysis. Adjusting for model risk under this model provides sufficient capital to cover all losses during our almost twenty five year period, except the Black Monday 1987 for equity and the event of Britain leaving EMU in 1992, both events being entangled with political decisions. At the same time, a system based on capital requirement adjusted for model risk under the NIG model would have had sufficient capital to cover the large losses during the dot-com crisis of 1999-2001 and also the Lehman Brothers collapse in 2008, both events emerging from an endogeneized market risk. Furthermore, the expected losses risk measure as quantified through VaR performed very well across time overall to cover *non-extreme* losses, confirming current views in the literature supporting this measure of risk, see Davis (2014). For unexpected losses we need an additional risk measure to work in tandem with VaR, and the measure we advocated, MS, is flexible enough to capture unexpected losses.

Our findings lead us to argue that, when imposing appropriate assumptions on the distribution of asset returns, our measures of unexpected losses adjusted for parameter risk estimation are a valuable starting point in order to define a capital requirements scheme that allows banks to hold a reasonable buffer of reserves and to employ at most the available resources in productive activities.

7.2 Social Significance of Miscalculations

The BCBS excessive capital requirements induce losing interest on money locked in buffer accounts. The analysis presented here shows the costs related to holding capital requirements computed according to the BCBS regulation[33] versus the calculations of capital requirements determined on the basis of our measures of losses. For simplicity we assume an initial investment, buy and hold, of \$100 in the S&P500 and in the GBP. These costs are calculated according to the following procedure: each day we compute the difference between BCBS capital requirements and the maximum between the realized 10-day losses and our measures for losses. When the difference is negative there is a drawdown from the buffer account to cover realized losses. This happens rarely. Most of the time the difference is positive, and for all those days, we calculate the interests accrued on this amount, with daily compounding, with respect to the US dollar 3-month deposit middle rate with continuous compounding. The final numbers show the total amount of interest that could have been earned from a money market account paying interest at the risk-free rate.

Table 7 provides the summary of these calculations for all five models and it also shows the number of times in which the realized 10-day losses exceed the BCBS capital requirements given only by the first term of the formula (23). Over the period 02/01/1984−10/07/2014 for the USD/GBP rate the BCBS capital requirement

[33]Note that only the first term in formula (23) is used. Hence, in reality the costs are even higher.

threshold (unstressed) is not breached by any 10-day loss. On the equity side, for S&P500, the BCBS capital requirement threshold (unstressed) was insufficient to cover 10-day losses only on four days, for all models except the NIG when there is only one instance of a loss higher than the capital buffer. The GARCH models seem to be the most exposed to large losses, more conservative risk managers perhaps being tempted by the nonparametric historical simulation approach. The level of cumulative lost interest varies between 21.90% and 41.86% for equity and between 24.63% and 26.62% for foreign exchange.

When comparing the performance of risk measures from a frequency coverage perspective we can observe that the NIG model does a great job. Great savings can be achieved using this model, roughly 40% in equity space and 25% in foreign exchange space. Given that over the period of our study the stock market capitalization increased from 3 trillion USD to almost 20 trillion USD, we can see that a lot of capital can be saved and be used in other parts of the economy. The percentage is smaller for the foreign exchange but, according to the BIS, as of January 2014, the foreign-exchange trading increased to an average of $5.3 trillion a day.[34] Great savings can be achieved by safeguarding against risk more efficiently in the financial markets based on robust tested methods. In addition, as Kashyap et al. (2010) and Hanson et al. (2011) observed, given the intensity of competition in financial services, higher capital requirements will dislocate a larger share of intermediation into the shadow banking, which will tilt the plain level field in financial sector.

How much money a bank should keep as a buffer against future losses is far from a trivial questions. Allen et al. (2011) highlighted that although banks had between 1990s and 2000s capital levels well above the regulatory mimimums, the impact of the financial crises that started with the subprime crisis of 2007 points out that perhaps banks were in fact undercapitalized vis-a-vis a social safety level. A possible solution as suggested by Allen et al. (2011) could be for banks to manage risk not by holding

[34]This figure includes all currencies.

capital but by increasing their loan rates.

[Table 7 about here.]

8 Summary and Concluding Remarks

In this paper we propose a framework for incorporating model risk related to the estimation of some standard and some new market risk monitoring and validation tools. We extended the approach proposed by Kerkhof et al. (2010) by using quantile risk measures for which one can derive confidence intervals. In addition, we employ the distance between MS and VaR as a reliable measure of unexpected losses that should be covered by capital reserves. Our improved methodology delivers extended risk measures embedding model risk that are ratified through several well-known backtests. We propose a new backtesting tool that can be used to validate both the risk measure and the risk model with respect to unexpected losses.

Important crash-like events such as Black Monday 1987 or Black Wednesday 1992 when Britain's exited from EMU led to an immediate ballooning of risk measures and to an overhang level of high risk for a significant period of time. These losses were caused by political events to the equity and foreign exchange markets and one cannot expect VaR, as a measure of expected losses, to capture them endogenously. The dot-com bubble crisis of 1999-2001 was an endogenous risk event of a longer duration. Our MS calculations signalled correctly an imminent market crash.

Our model validation tools can be helpful to investment banks, hedge funds but also to regulators in order to differentiate among competing risk models. The simple gaussian model is totally inadequate to deal with unexpected or extreme losses. At the same time the NIG model is clearly superior to other common approaches and adjusting for model risk may also improve backtesting overall. In addition, the NIG model produced higher levels of risk forecasts than those produced by the gaussian model, indicating that using the latter may grossly underestimate risk. Furthermore, for the NIG model is more sensitive to tail risk as the associated MS and ES time series suggest. We conjecture that skewness should play an important role in risk management and *different* risk measures may be used for different asset classes.

VaR calculations are still important and they cover, as they should by definition, expected losses while tail measures such as MS (or ES) are meant to gauge unexpected losses. ES is a risk measure that can be influenced by extreme losses caused by rogue trading, for example, and it is more difficult to compute and backtest. MS is a tail risk measure that is easier to calculate than ES and is not subjected to the estimation error of VaR as a threshold of expected versus unexpected losses. Taking into account that ES is difficult to backtest in general while MS being a quantile-type measure is not, our recommendation is to focus on the VaR and MS as risk measures and allow banks to select their internal models that should be validated on their portfolios. No single model seems to pass all backtests applied for the two main markets, equity and foreign. The NIG model has had the best performance overall associated with the data included in this paper. It is likely that there is no model that will pass all backtests all the time. Thus, model selection and adjusting for parameter estimation is a very important exercise in risk management that must be carried out internally by banks on a regular basis.

In addition, comparing capital requirements recommended by the Basel 3 regulation with our estimates of losses adjusted for parameter risk estimation, we find that the former are *excessively* conservative, whilst the latter is a flexible alternative, not difficult to implement and interpret. In our view, capital reserves computed on the basis of our measures of unexpected losses adjusted for parameter risk estimation allow to deal better with the trade-off between the threshold related to loss exceeding capital requirements and the cost of impeding banks in their operations by charging high regulatory reserves.

Our criticism of the current set of regulations imposed by regulators is not singular (see Kerkhof et al., 2010). Vallasca and Hagendorff (2013) evaluated the risk sensitivity of minimum capital requirements and found evidence of ill-calibration to a market measure of bank portfolio risk. The amount of capital that may get locked over time due to extra-conservative regulatory measures may reach trillions of dollars as an order of magnitude. The society will benefit from these policies by avoiding a repetition of the recent crises, but it may also suffer from lack of liquidity available in other non-banking areas.

Dimson and Marsh (1995) compared the initial three main approaches for computing capital requirements: the comprehensive approach of the U.S. Securities and Exchange Commission, the building-block approach required by the European Community, and the portfolio approach proposed by the United Kingdom. Relative to a large sample of equity trading books from U.K. they show that the portfolio approach is by far superior to the other two approaches, increasing capital requirements when the risk is high and lowering them when the risk was low. On the other hand Jokivuolle et al. (2014) provided a rationale for maintaining risk-based capital requirements higher in good times and lowering them in bad times. The capital requirements BCBS methodology may require a more thorough analysis of its design.

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Figure 1: Quantiles used in order to derive the 95% distribution-free confidence intervals for VaR and MS

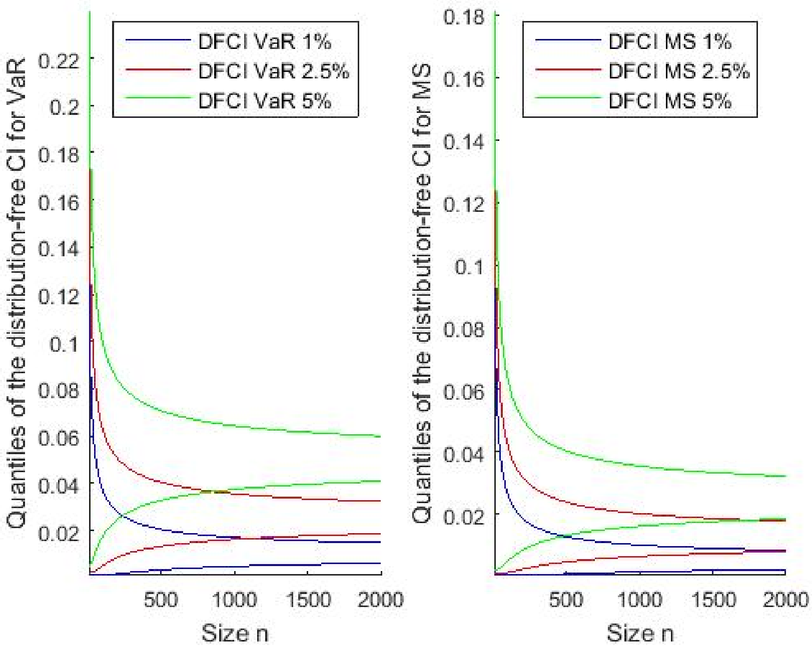
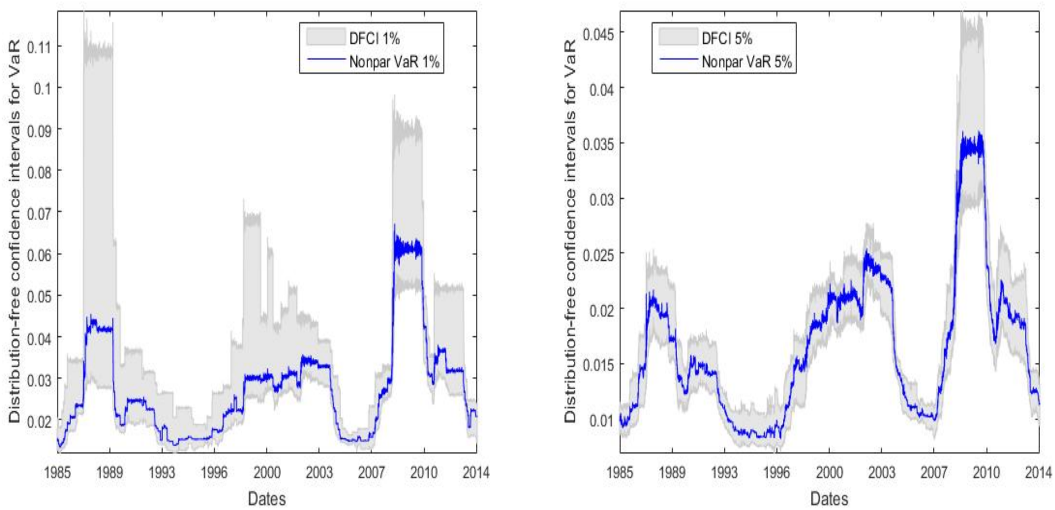


Figure 2: Time series of the nonparametric VaR and the 95% distribution-free confidence intervals for VaR of a long position on the S&P 500 and a long position on the USD/GBP exchange rate. Risk measures and their bounds are expressed as a proportion of spot prices and are computed at 1% and 5% level.

(a) Nonparametric VaR and 95% DFCI for S&P 500



(b) Nonparametric VaR and 95% DFCI for USD/GBP

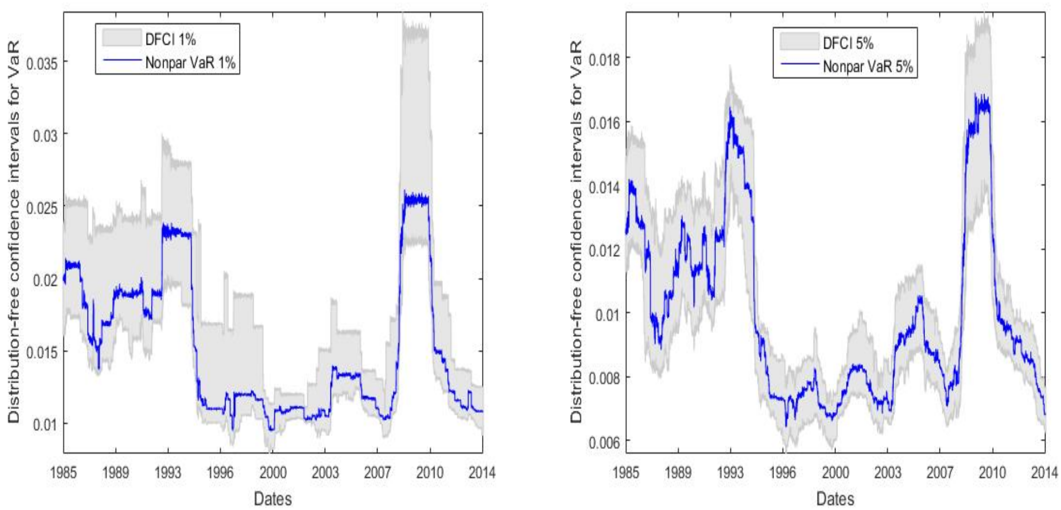
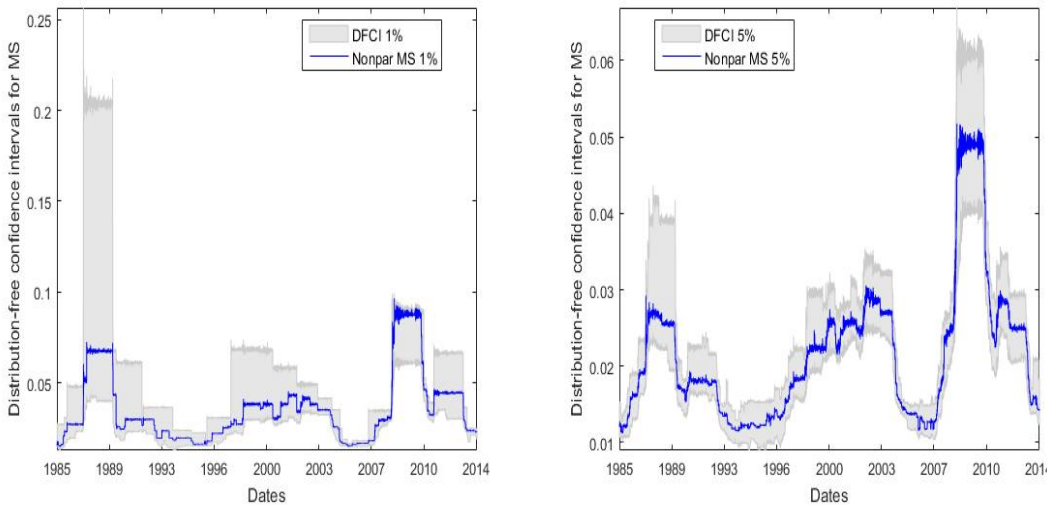


Figure 3: Time series of the nonparametric MS and the 95% distribution-free confidence intervals for MS of a long position on the S&P 500 and a long position on the USD/GBP exchange rate. Risk measures and their bounds are expressed as a proportion of spot prices and are computed at 1% and 5% level.

(a) Nonparametric MS and 95% DFCI for S&P 500



(b) Nonparametric MS and 95% DFCI for USD/GBP

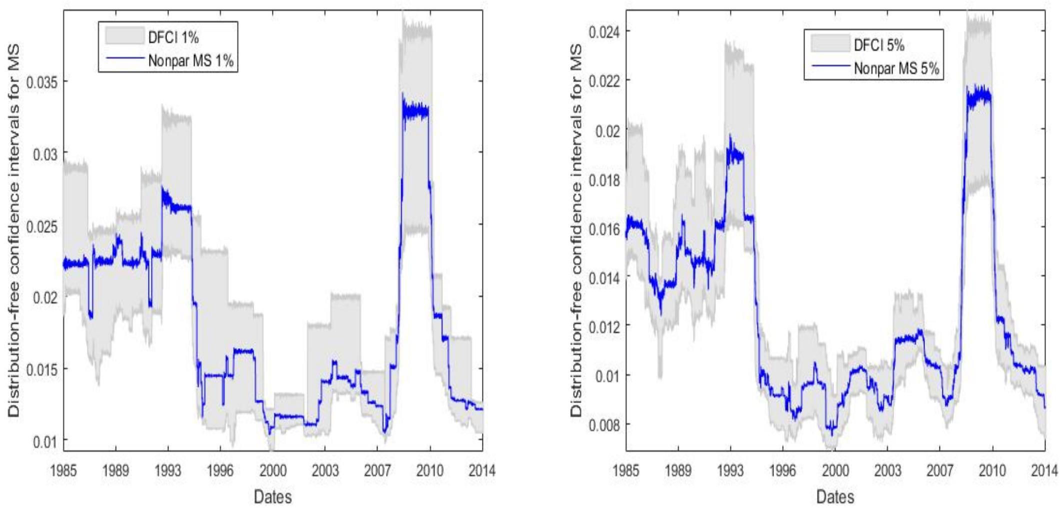


Figure 4: Time series of the nonparametric ES and the nonparametric MS of a long position on the S&P 500 (left side) and a long position on the USD/GBP exchange rate (right side). Risk measures are expressed as a proportion of spot prices and are computed at 1% level, 2.5% level and 5% level.

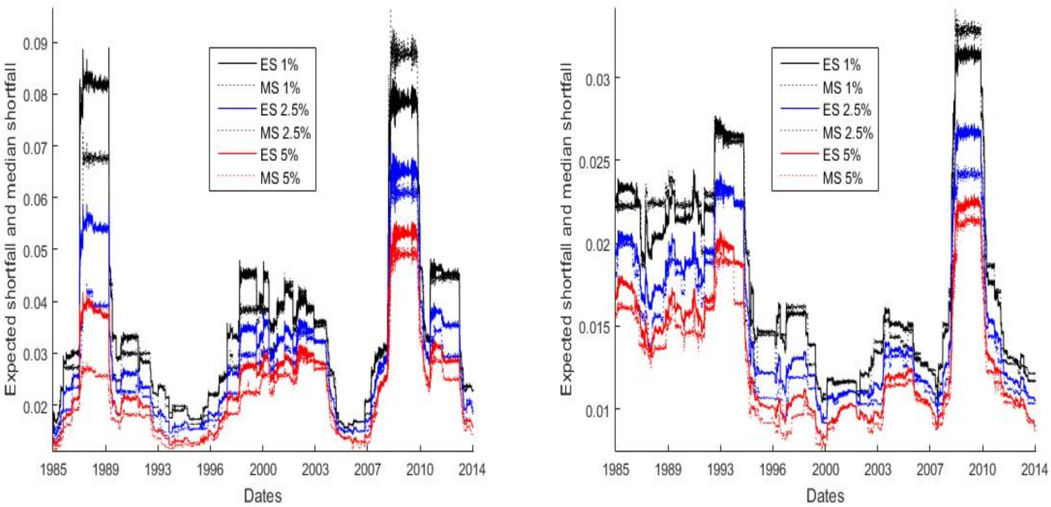
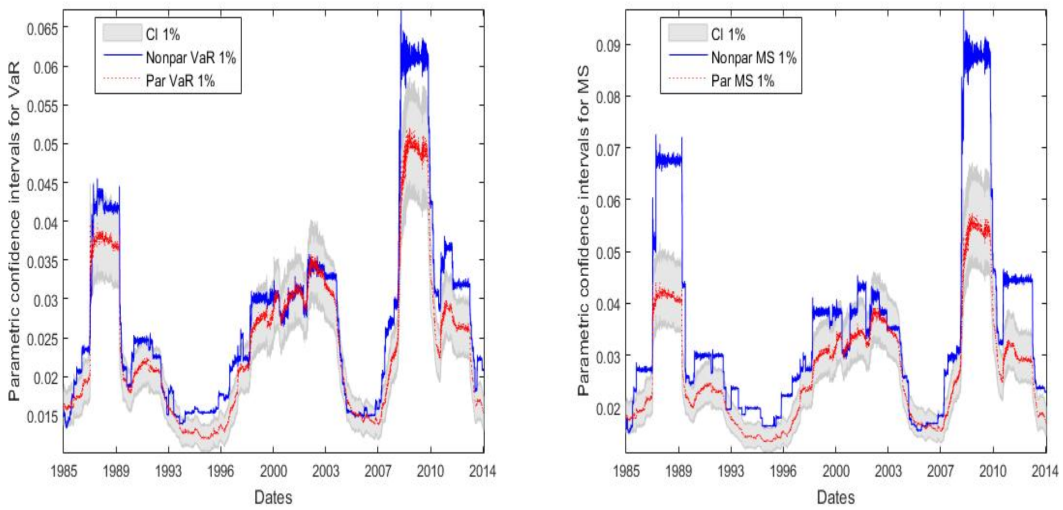


Figure 5: Time series of the parametric VaR, the parametric MS and the related 95% confidence intervals for a long position on the S&P 500 and a long position on the USD/GBP exchange rate. Calculations are done assuming a gaussian distribution for the returns of the asset. Risk measures and their bounds are expressed as a proportion of spot prices and are computed at 1% level.

(a) Parametric VaR and MS with 95% CI for S&P 500



(b) Parametric VaR and MS with 95% CI for USD/GBP

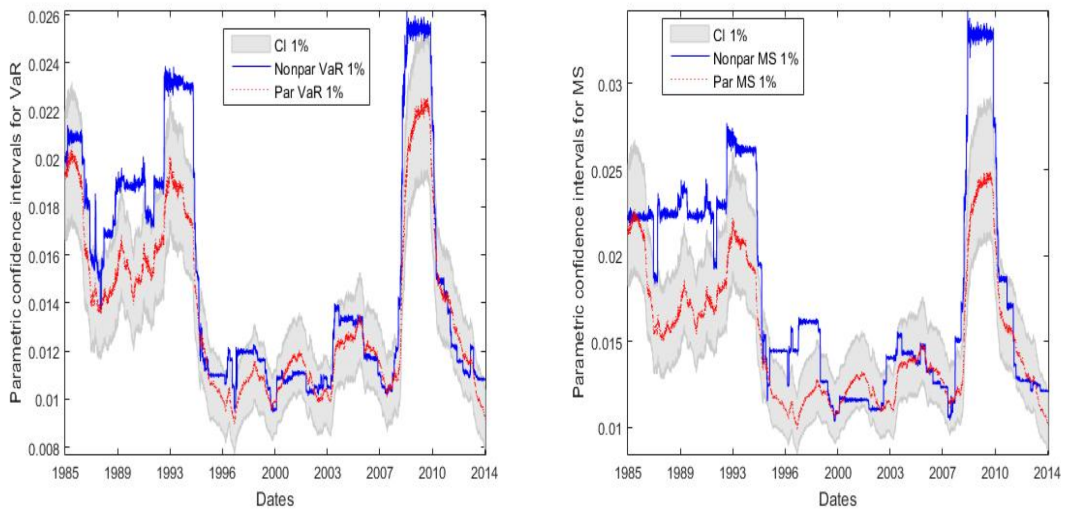
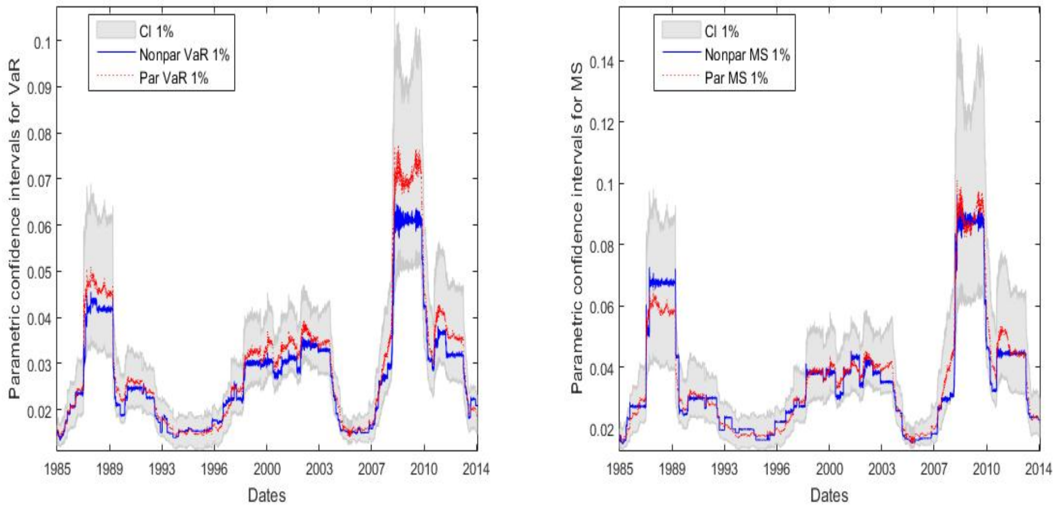


Figure 6: Time series of the parametric VaR, the parametric MS and the related 95% confidence intervals for a long position on the S&P 500 and a long position on the USD/GBP exchange rate. Calculations are done assuming a NIG distribution for the returns of the asset. Risk measures and their bounds are expressed as a proportion of spot prices and are computed at 1% level.

(a) Parametric VaR and MS with 95% CI for S&P 500



(b) Parametric VaR and MS with 95% CI for USD/GBP

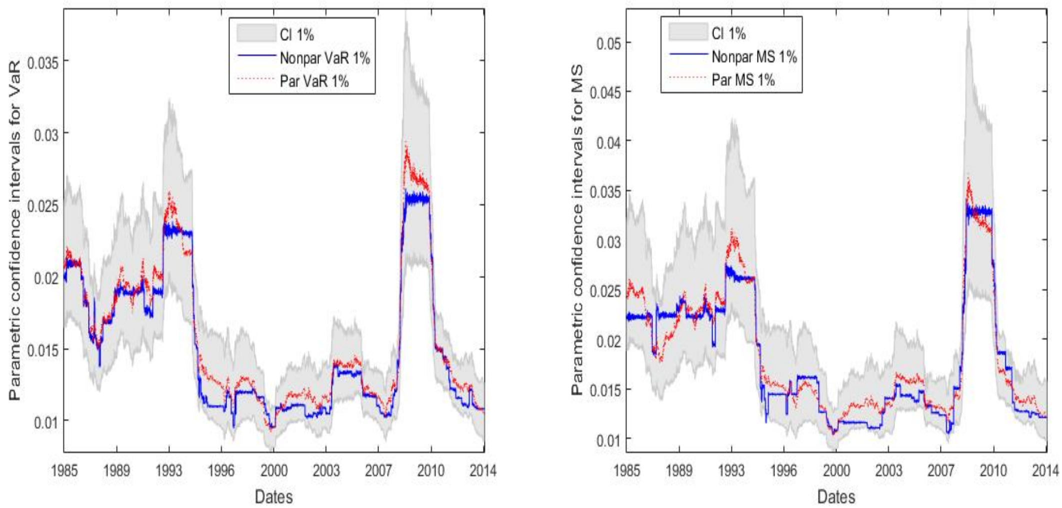
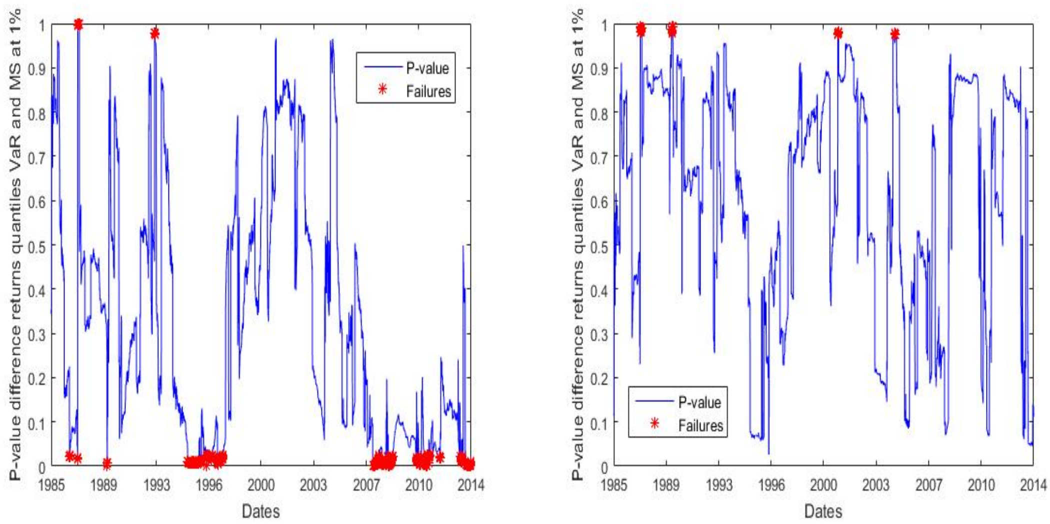


Figure 7: Time series of the p-values of the difference between the quantiles of the returns empirical distribution used to calculate VaR and MS at 1% for the S&P 500 and the USD/GBP exchange rate. P-values are calculated assuming that the true distribution of financial returns is gaussian (left) and NIG (right). Marks highlight failures, namely the dates in which the computed probability to overcome the estimated difference between the quantiles of the returns used to calculate the nonparametric VaR and the nonparametric MS is rejected at 5%.

(a) P-values VaR-MS at 1% under gaussian (left) and NIG (right) for S&P 500



(b) P-values VaR-MS at 1% under gaussian (left) and NIG (right) for USD/GBP

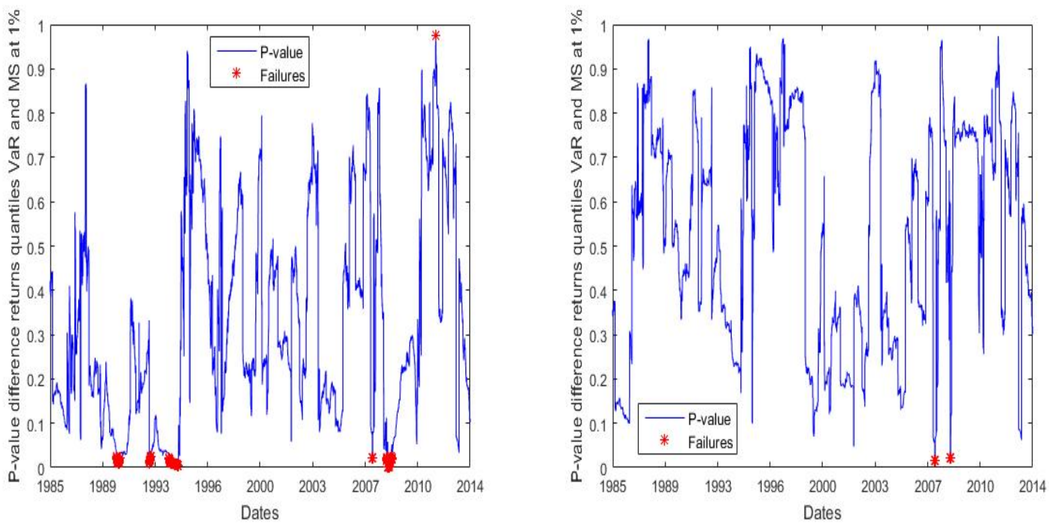
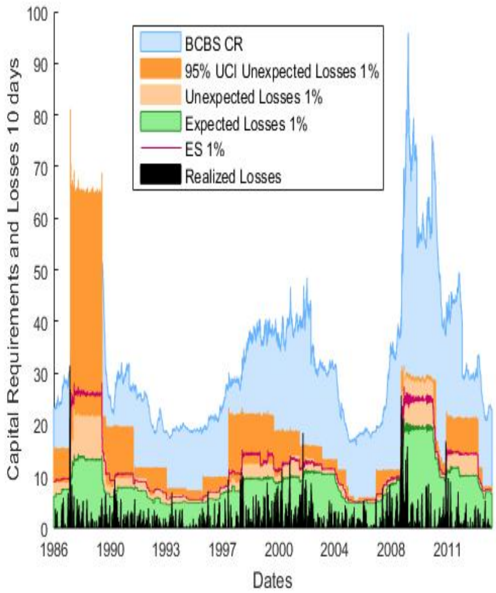
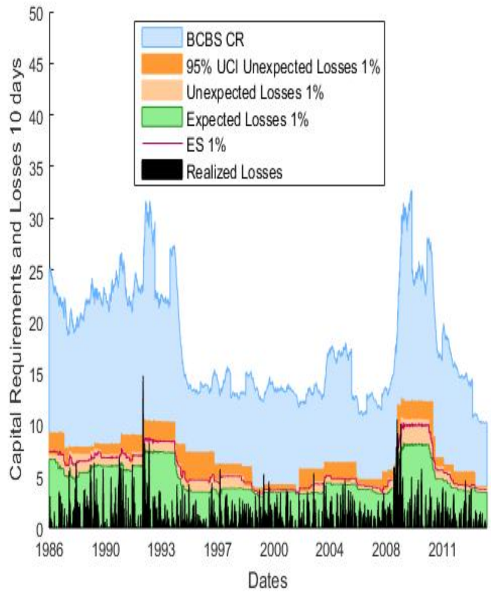


Figure 8: Time series of two-week losses, expected losses computed according to the nonparametric VaR, unexpected losses calculated on the basis of the nonparametric MS, upper part of the 95% distribution-free confidence interval – (a) and (b) – and the 95% parametric confidence interval – (c) and (d) – for the nonparametric MS, the nonparametric ES, capital requirements computed according to BCBS (only first term in formula (23)) for the nonparametric VaR – (a) and (b) – and the parametric VaR – (c) and (d) –, for an investment of $100 in the S&P 500 and in the USD/GBP exchange rate. The 95% confidence interval for the nonparametric MS, the nonparametric ES and the BCBS capital requirements are calculated assuming a nonparametric i.i.d model – (a) and (b) – and a NIG distribution – (c) and (d) – for the returns of the asset.

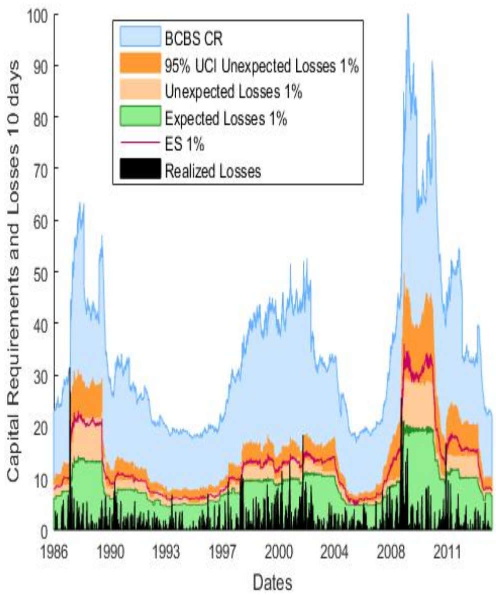
(a) S&P 500 under nonparametric



(b) USD/GBP under nonparametric



(a) S&P 500 under NIG



(b) USD/GBP under NIG

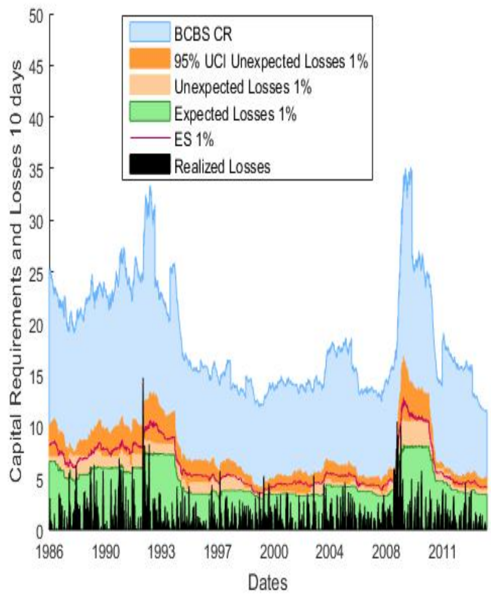


Table 1: Meta-analysis comparison of main risk measures VaR, MS and ES.

Property	VaR	MS	ES	Source
Sub-additivity	X	X	$\surd$	Artzner et al. (1999),McNeil et al. (2015),Acerbi and Tasche (2002)
Homogeneity	$\surd$	$\surd$	$\surd$	Artzner et al. (1999),McNeil et al. (2015),Acerbi and Tasche (2002)
Risk-free reduction	$\surd$	$\surd$	$\surd$	Artzner et al. (1999),McNeil et al. (2015),Acerbi and Tasche (2002)
Monotonicity	$\surd$	$\surd$	$\surd$	Artzner et al. (1999),McNeil et al. (2015),Acerbi and Tasche (2002)
Comonotonic	$\surd$	$\surd$	X	Emmer et al. (2015)
Coherent under elliptical family of distributions	$\surd$	$\surd$	$\surd$	So and Wong (2012)
Sub-additivity in special cases	$\surd$	$\surd$	$\surd$	Dhaene et al. (2006),Danielsson et al. (2013),Ibragimov (2009)
Coherent with respect to a stochastic order	$\surd$	$\surd$	$\surd$	Rémillard (2013)
Elicitability	$\surd$	$\surd$	X	Gneiting (2011),Ziegel (2014)
Backtesting possible	$\surd$	$\surd$	$\surd$	Gneiting (2011), Escanciano and Olmo (2011), Pérignon and Smith (2010)
Sensitive to magnitude of losses in the tail	X	$\surd$	$\surd$	Colletaz et al. (2013),Danielsson et al. (2013),Dhaene and Salahnejhad (2015)
Irrelevance of positive gains	$\surd$	$\surd$	$\surd$	This paper
Asymptotic Monotonicity of specific risk	$\surd$	$\surd$	$\surd$	This paper
Consistent	$\surd$	$\surd$	X	Davis (2014)
Robust	$\surd$	$\surd$	X	Kou et al. (2013),Kou et al. (2013)
Robustness to model perturbation	$\surd$	$\surd$	X	Cont et al. (2010)
Computationally inexpensive	$\surd$	$\surd$	X	Yamai and Yoshiba (2005)

Table 2: FOEL Test, Kupiec's POF Test, Christoffersen's Independence Test, Christoffersen-Pelletier's Independence Test, Christoffersen's Conditional Coverage Test, Berkowitz-Christoffersen-Pelletier Conditional Coverage Test, Engle-Manganelli's Dynamic Quantile Conditional Coverage Test for VaR based risk measures related to a long position on the S&P 500. Risk measures are the nonparametric VaR, the VaR computed assuming a Normal distribution and a NIG distribution for the returns of the asset, the upper bound of the distribution-free confidence interval for the nonparametric VaR, the upper bound of the parametric confidence intervals for the nonparametric VaR estimated assuming a Normal distribution and a NIG distribution for the returns of the asset, and the VaR calculated assuming a GARCH(1,1)-Normal model and a GJR-GARCH(1,1)-Normal model for the returns of the asset. PUB and DFUB denote, respectively, the parametric upper bound and the distribution-free upper bound determined with order statistics.

Model	**Critical level**	**FOEL T-stat**	**Ku POF T-stat**	**Ch I T-stat**	**Ch-Pe I T-stat**	**Ch CC T-stat**	**Be-Ch-Pe CC T-stat**	**DQ CC T-stat**
Nonparametric VaR	1%	5.192	22.827	5.879*	58.768	28.706	80.395	27.762
Normal VaR	1%	10.900	87.618	23.534	104.980	111.152	189.782	118.434
NIG VaR	1%	3.328	9.902	0.212***	49.362	10.114	58.513	31.930
DFUB VaR	1%	-4.826***	29.400	2.181***	19.290	31.581	50.115	38.689
Normal PUB VaR	1%	5.658	26.766	10.589	74.822	37.355	100.270	35.322
NIG PUB VaR	1%	-3.778***	16.930	1.381***	18.765	18.311	36.720	28.878
GARCH-N VaR	1%	10.667	84.334	0.430***	3.407**	84.765	84.999	46.003
GJR-GARCH-N VaR	1%	10.201	77.916	0.064***	0.467***	77.979	75.788	57.169
Nonparametric VaR	2.5%	2.370	5.338*	22.867	97.563	28.205	102.448	32.146
Normal VaR	2.5%	5.266	24.849	24.104	103.597	48.954	127.108	77.221
NIG VaR	2.5%	2.593	6.358*	19.444	93.689	25.802	99.538	32.037
DFUB VaR	2.5%	-3.718***	15.210	16.815	75.267	32.026	90.767	41.221
Normal PUB VaR	2.5%	0.588***	0.344***	16.281	104.912	16.624	105.169	22.300
NIG PUB VaR	2.5%	-3.866***	16.521	11.198	70.402	27.720	87.215	45.600
GARCH-N VaR	2.5%	6.379	35.726	1.597***	0.453***	37.323	34.401	28.538
GJR-GARCH-N VaR	2.5%	6.305	34.946	0.993***	0.057***	35.939	33.256	27.243
Nonparametric VaR	5%	1.481**	2.152***	14.557	80.876	16.709	82.775	85.967
Normal VaR	5%	0.949***	0.893***	13.753	93.016	14.646	93.773	72.028
NIG VaR	5%	2.066*	4.146*	15.096	87.588	19.242	91.324	95.202
DFUB VaR	5%	-5.326***	31.142	24.466	89.968	55.608	120.330	56.600
Normal PUB VaR	5%	-3.518***	13.123	19.454	102.018	32.577	114.919	50.594
NIG PUB VaR	5%	-4.901***	26.146	24.441	103.593	50.587	129.116	50.982
GARCH-N VaR	5%	2.279*	5.027*	1.079***	0.297***	6.106*	4.848**	43.911
$\boxed{GJR-GARCH-N\ \ VaR}$	5%	2.119*	4.359*	0.229***	0.940***	4.588***	4.872**	19.903

*, ** and *** denotes the model that passes the test at 1% level, 5% level and 10% level

Table 3: FOEL Test, Kupiec's POF Test, Christoffersen's Independence Test, Christoffersen-Pelletier's Independence Test, Christoffersen's Conditional Coverage Test, Berkowitz-Christoffersen-Pelletier Conditional Coverage Test, Engle-Manganelli's Dynamic Quantile Conditional Coverage Test for MS based risk measures related to a long position on the S&P 500. Risk measures are the nonparametric MS, the MS computed assuming a Normal distribution and a NIG distribution for the returns of the asset, the upper bound of the distribution-free confidence interval for the nonparametric MS, the upper bound of the parametric confidence intervals for the nonparametric MS computed assuming a Normal distribution and a NIG distribution for the returns of the asset, and the MS calculated assuming a GARCH(1,1)-Normal model and a GJR-GARCH(1,1)-Normal model for the returns of the asset. PUB and DFUB denote, respectively, the parametric upper bound and the distribution-free upper bound determined with order statistics.

Model	**Critical level**	**FOEL T-stat**	**Ku POF T-stat**	**Ch I T-stat**	**Ch-Pe I T-stat**	**Ch CC T-stat**	**Be-Ch-Pe CC T-stat**	**DQ CC T-stat**
Nonparametric MS	1%	4.073	13.812	0.362***	36.800	14.174	49.524	22.181
Normal MS	1%	14.919	135.814	9.595	81.569	145.409	213.802	209.497
$\boxed{NIG\ \ MS}$	1%	2.594	5.947*	0.718***	25.027	6.666*	30.253	6.062***
DFUB Nonpar MS	1%	-3.322***	13.841	4.686*	15.741	18.527	31.153	33.425
PUB Normal MS	1%	6.867	35.610	1.189***	63.629	36.798	97.509	46.975
PUB NIG MS	1%	-3.486***	15.473	4.928*	2.312***	20.401	19.476	39.761
GARCH-N MS	1%	11.961	93.597	0.962***	9.400	94.559	100.120	35.156
GJR-GARCH-N MS	1%	10.482	74.677	0.114***	1.122***	74.790	73.261	20.526
Nonparametric MS	2.5%	4.482	17.604	16.815	75.267	34.419	91.867	35.872
Normal MS	2.5%	10.116	78.715	17.280	107.029	95.994	183.035	110.271
NIG MS	2.5%	3.439	10.658	9.984	68.861	20.643	78.772	34.970
DFUB Nonpar MS	2.5%	-3.968***	18.438	0.626***	33.779	19.064	53.090	37.627
PUB Normal MS	2.5%	4.378	16.840	13.922	91.298	30.762	107.160	33.060
PUB NIG MS	2.5%	-3.968***	18.438	0.626***	24.013	19.064	43.323	25.975
GARCH-N MS	2.5%	9.490	70.182	0.444***	1.711***	70.626	69.403	52.372
GJR-GARCH-N MS	2.5%	10.012	77.265	0.936***	0.739***	78.201	75.333	47.998
Nonparametric MS	5%	2.370	5.338*	22.867	97.563	28.205	102.448	32.146
Normal MS	5%	5.266	24.849	24.104	103.597	48.954	127.108	77.221
NIG MS	5%	2.593	6.358*	19.444	93.689	25.802	99.538	32.037
DFUB Nonpar MS	5%	-3.718***	15.210	16.815	75.267	32.026	90.767	41.220
PUB Normal MS	5%	0.588***	0.344***	16.281	104.912	16.624	105.169	22.300
PUB NIG MS	5%	-3.866***	16.521	11.198	70.402	27.720	87.215	45.600
GARCH-N MS	5%	6.379	35.726	1.597***	0.453***	37.323	34.401	28.538
GJR-GARCH-N MS	5%	6.305	34.946	0.993***	0.057***	35.939	33.256	27.243

*, ** and *** denotes the model that passes the test at 1% level, 5% level and 10% level

Table 4: FOEL Test, Kupiec's POF Test, Christoffersen's Independence Test, Christoffersen-Pelletier's Independence Test, Christoffersen's Conditional Coverage Test, Berkowitz-Christoffersen-Pelletier Conditional Coverage Test, Engle-Manganelli's Dynamic Quantile Conditional Coverage Test for VaR based risk measures related to a long position on the USD/GBP exchange rate. Risk measures are the nonparametric VaR, the VaR computed assuming a Normal distribution and a NIG distribution for the returns of the asset, the upper bound of the distribution-free confidence interval for the nonparametric VaR, the upper bound of the parametric confidence intervals for the nonparametric VaR computed assuming a Normal distribution and a NIG distribution for the returns of the asset, and the VaR calculated assuming a GARCH(1,1)-Normal model and a GJR-GARCH(1,1)-Normal model for the returns of the asset. PUB and DFUB denote, respectively, the parametric upper bound and the distribution-free upper bound determined with order statistics.

Model	**Critical level**	**FOEL T-stat**	**Ku POF T-stat**	**Ch I T-stat**	**Ch-Pe I T-stat**	**Ch CC T-stat**	**Be-Ch-Pe CC T-stat**	**DQ CC T-stat**
Nonparametric VaR	1%	2.047*	3.902*	7.417	36.735	11.319	40.182	12.243
Normal VaR	1%	6.008	29.894	27.237	79.559	57.132	108.046	26.493
NIG VaR	1%	-0.050***	0.002***	4.046*	45.573	4.048***	45.601	10.219***
DFUB VaR	1%	-5.875***	46.864	0.000***	5.957*	46.864	54.782	34.431
Normal PUB VaR	1%	0.532***	0.279***	9.985	60.615	10.264	60.785	16.147*
NIG PUB VaR	1%	-4.826***	29.400	0.000***	14.756	29.400	45.581	28.154
GARCH-N VaR	1%	4.377	16.597	2.383***	0.088***	18.981	15.685	39.594
GJR-GARCH-N VaR	1%	4.726	19.162	0.028***	0.437***	19.189	18.513	51.960
Nonparametric VaR	2.5%	0.811***	0.649***	18.106	54.560	18.755	55.083	28.736
Normal VaR	2.5%	4.226	16.333	34.069	56.141	50.402	71.496	23.355
NIG VaR	2.5%	1.108***	1.201***	19.684	50.047	20.885	51.068	21.023
DFUB VaR	2.5%	-6.093***	44.046	8.333	38.049	52.380	82.344	59.981
Normal PUB VaR	2.5%	-2.827***	8.573	26.825	72.569	35.398	81.408	30.581
NIG PUB VaR	2.5%	-6.465***	50.223	6.258*	37.926	56.481	88.381	64.028
GARCH-N VaR	2.5%	3.261	9.916	0.497***	0.046***	10.413	9.276	16.619*
GJR-GARCH-N VaR	2.5%	3.558	11.733	0.019***	0.313***	11.751	11.274	20.041
Nonparametric VaR	5%	-0.487***	0.236***	36.135	44.384	36.371	44.660	31.553
Normal VaR	5%	-0.114***	0.012***	38.125	48.970	38.138	48.996	24.202
NIG VaR	5%	0.471***	0.222***	34.195	43.916	34.417	44.082	23.520
DFUB VaR	5%	-6.656***	49.972	24.608	44.748	74.580	93.343	88.400
Normal PUB VaR	5%	-4.688***	23.826	38.228	55.374	62.054	78.651	56.754
NIG PUB VaR	5%	-5.858***	38.077	30.122	45.237	68.199	82.314	69.294
$\boxed{GARCH-N\ \ VaR}$	5%	1.056***	1.102***	6.056*	0.006***	7.158*	0.951***	11.146**
GJR-GARCH-N VaR	5%	1.853*	3.348**	4.261*	0.360***	7.609*	3.359***	16.112*

*, ** and *** denotes the model that passes the test at 1% level, 5% level and 10% level

Table 5: FOEL Test, Kupiec's POF Test, Christoffersen's Independence Test, Christoffersen-Pelletier's Independence Test, Christoffersen's Conditional Coverage Test, Berkowitz-Christoffersen-Pelletier Conditional Coverage Test, Engle-Manganelli's Dynamic Quantile Conditional Coverage Test for MS based risk measures related to a long position on the USD/GBP exchange rate. Risk measures are the nonparametric MS, the MS computed assuming a Normal distribution and a NIG distribution for the returns of the asset, the upper bound of the distribution-free confidence interval for the nonparametric MS, the upper bound of the parametric confidence intervals for the nonparametric MS computed assuming a Normal distribution and a NIG distribution for the returns of the asset, and the MS calculated assuming a GARCH(1,1)-Normal model and a GJR-GARCH(1,1)-Normal model for the returns of the asset. PUB and DFUB denote, respectively, the parametric upper bound and the distribution-free upper bound determined with order statistics.

Model	**Critical level**	**FOEL T-stat**	**Ku POF T-stat**	**Ch I T-stat**	**Ch-Pe I T-stat**	**Ch CC T-stat**	**Be-Ch-Pe CC T-stat**	**DQ CC T-stat**
Nonparametric MS	1%	1.279***	1.536***	0.000***	16.812	1.536***	17.982	10.774**
Normal MS	1%	7.688	43.508	8.927	59.650	52.436	101.245	69.075
$\boxed{NIG\ MS}$	1%	-0.035***	0.001***	1.791***	21.925	1.792***	21.965	4.960***
DFUB Nonpar MS	1%	-3.979***	21.157	0.000***	1.034***	21.157	24.294	16.060*
PUB Normal MS	1%	1.608**	2.389***	1.044***	35.668	3.433***	37.598	5.603***
PUB NIG MS	1%	-4.308***	25.702	0.000***	1.376***	25.702	29.516	22.335
GARCH-N MS	1%	6.045	28.351	0.000***	2.064***	28.351	28.871	55.479
GJR-GARCH-N MS	1%	6.374	31.175	0.000***	0.843***	31.175	30.399	65.288
Nonparametric MS	2.5%	1.144***	1.263***	8.333	38.049	9.596	39.083	12.157**
Normal MS	2.5%	5.108	22.497	25.303	67.102	47.800	88.433	28.500
NIG MS	2.5%	0.100***	0.010***	7.072	43.520	7.082*	43.520	17.330
DFUB Nonpar MS	2.5%	-6.055***	48.081	0.000***	9.412	48.081	59.020	34.226
PUB Normal MS	2.5%	-0.213***	0.045***	7.595	61.323	7.640*	61.422	19.068
PUB NIG MS	2.5%	-5.638***	40.640	1.617***	22.661	42.258	64.669	32.504
GARCH-N MS	2.5%	4.795	19.985	1.832***	1.301***	21.817	20.201	34.041
GJR-GARCH-N MS	2.5%	5.108	22.497	0.212***	0.004***	22.709	21.336	44.541
Nonparametric MS	5%	0.811***	0.649***	18.106	54.560	18.755	55.083	28.736
Normal MS	5%	4.226	16.333	34.069	56.141	50.402	71.496	23.355
NIG MS	5%	1.108***	1.201***	19.684	50.047	20.885	51.068	21.023
DFUB Nonpar MS	5%	-6.093***	44.046	8.333	38.049	52.380	82.344	59.981
PUB Normal MS	5%	-2.827***	8.573	26.825	72.569	35.398	81.408	30.581
PUB NIG MS	5%	-6.465***	50.223	6.258*	37.926	56.481	88.381	64.028
GARCH-N MS	5%	3.261	9.916	0.497***	0.046***	10.413	9.276	16.619*
GJR-GARCH-N MS	5%	3.558	11.733	0.019***	0.313***	11.751	11.274	20.041

*, ** and *** denotes the model that passes the test at 1% level, 5% level and 10% level

Table 6: Pérignon and Smith Multivariate Unconditional Coverage Test for VaR based and MS based risk measures related to a long position on the S&P 500 and a long position on the USD/GBP exchange rate. Risk measures are the nonparametric VaR/MS, the VaR/MS computed assuming a Normal distribution and a NIG distribution for the returns of the asset, the upper bound of the distribution-free confidence interval for the nonparametric VaR/MS, the upper bound of the parametric confidence intervals for the nonparametric VaR/MS estimated assuming a Normal distribution and a NIG distribution for the returns of the asset, and the VaR/MS calculated assuming a GARCH(1,1)-Normal model and a GJR-GARCH(1,1)-Normal model for the returns of the asset. PUB and DFUB denote, respectively, the parametric upper bound and the distribution-free upper bound determined with order statistics. For each risk measure the test is performed on a vector of estimates computed at three critical levels, namely 1%, 2.5% and 5%.

	M. U. C. T-stat	
Model	**S&P 500**	**USD/GBP**
Nonparametric VaR	21.077	7.458**
Normal VaR	106.239	51.128
NIG VaR	5.905***	1.281***
DFUB VaR	53.808	78.210
Normal PUB VaR	84.198	42.168
NIG PUB VaR	38.250	63.875
GARCH-N VaR	89.517	17.861
GJR-GARCH-N VaR	84.404	16.492
Nonparametric MS	18.670	1.126***
Normal MS	139.060	40.567
NIG MS	8.801*	1.227***
DFUB Nonpar MS	17.627	40.117
PUB Normal MS	49.278	23.044
PUB NIG MS	25.613	59.461
GARCH-N MS	93.043	27.109
GJR-GARCH-N MS	85.844	29.941

*, ** and *** denotes the model that passes the test at 1% level, 5% level and 10% level

Table 7: Costs related to holding capital requirements computed according to the BCBS regulation (first term of formula formula (23)) with respect to holding capital requirements determined on the basis of our measures of unexpected losses for an investment of $100 in the S&P 500 and in the USD/GBP exchange rate over the time period 02/01/1984 − 10/07/2014. The cumulative interest daily time series is calculated using the middle rate on 3-months deposits determined on the Eurocurrency market is used as proxy for the US risk free rate. Evidence on the number of times in which the realized 10-day losses exceed BCBS capital requirements (first term of formula (23)) and our measures of unexpected losses is also reported.

	S&P 500	USD/GBP
Cumulative interest lost (Nonparametric)	21.90	25.91
No. of extra losses on our CR (Nonparametric)	20	24
No. of extra 10-day losses on BCBS CR (Nonparametric)	4	0
Cumulative interest lost (Normal)	38.88	24.63
No. of extra 10-day losses on our CR (Normal)	52	73
No. of extra 10-day losses on BCBS CR (Normal)	4	0
Cumulative interest lost (NIG)	39.16	25.92
No. of extra 10-day losses on our CR (NIG)	15	19
No. of extra 10-day losses on BCBS CR (NIG)	1	0
Cumulative interest lost (GARCH(1,1)-Normal)	40.96	26.44
No. of extra 10-day losses on our CR (GARCH(1,1))	87	113
No. of extra 10-day losses on BCBS CR (GARCH(1,1))	4	0
Cumulative interest lost (GJR-GARCH(1,1)-Normal)	41.86	26.62
No. of extra 10-day losses on our CR (GJR-GARCH(1,1))	88	119
No. of extra 10-day losses on BCBS CR (GJR-GARCH(1,1))	4	0

Appendix

Proofs of Properties of Risk Measures

Here we derive the cdf of the negative part of a random variable Y. Denoting by $Z = \min(Y, 0)$ and by F_{YZ} the joint cdf of (Y, Z)

$$F_{YZ}(y, z) = P(Y \leq y, Z \leq z) = P(Y \leq y, \min(Y, 0) \leq z)$$

If $z \geq 0$ then $F_{YZ}(y, z) = P(Y \leq y) = F_Y(y)$. If$z < 0$

$$\begin{aligned}
F_{YZ}(y, z) &= P(Y \leq y, \min(Y, 0) \leq z) \\
&= P(Y \leq y, \min(Y, 0) \leq z, Y > 0) + P(Y \leq y, \min(Y, 0) \leq z, Y \leq 0) \\
&= P(Y \leq y, 0 \leq z, X > 0) + P(Y \leq y, Y \leq z, Y \leq 0) \\
&= P(Y \leq \min(y, z, 0)) \\
&= F_Y(\min(y, z, 0)) = F_Y(\min(y, z))
\end{aligned}$$

Hence

$$F_{YZ}(y, z) = \begin{cases} F_Y(y), & z \geq 0; \\ F_Y(\min(y, z)), & z < 0. \end{cases}$$

Then, because $F_Z(z) = \lim_{y \to \infty} F_{YZ}(y, z)$ we get that

$$F_Z(z) = \begin{cases} 1, & z \geq 0; \\ F_Y(z), & z < 0. \end{cases}$$

Here we detail the proofs for some of the results mentioned in the paper with respect to the monotonicity of specific risk property requiring that if a position Y_1 is cloned into independent copies $Y_1, Y_2, \ldots, Y_n, \ldots$ then for any integers $0 < m \leq n$ we have that

$$\frac{1}{n}\rho(Y_1 + Y_2 + \ldots Y_n) \le \frac{1}{m}\rho(Y_1 + Y_2 + \ldots Y_m) \tag{24}$$

so more positions of the same kind should reduce the risk per unit of trade. Thus we prove the following result.

Proposition 8.1. Proof*:*

If m and n are large enough, for any $\alpha \in (0, 0.5)$, VaR_α and ES_α are both monotonic to specific risk.

The intuition for this result lies in the central limit theorem because if $Y_1, Y_2, \ldots, Y_n, \ldots$ are i.i.d with mean μ and variance σ^2, then $Y_1 + \ldots + Y_n \sim N(n\mu, n\sigma^2)$. It is known that for a gaussian distributed variable $Y \sim N(\mu, \sigma^2)$ $VaR_\alpha(Y) = -\mu - \sigma\Phi^{-1}(\alpha)$ and $ES_\alpha(Y) = -\mu + \frac{\sigma}{\alpha}\varphi[\Phi^{-1}(\alpha)]$. Then for any large $m < n$ such that the CLT theorem applies

$$\frac{1}{n}VaR_\alpha(Y_1 + \ldots Y_n) < \frac{1}{m}VaR_\alpha(Y_1 + \ldots + Y_m)$$

and

$$\frac{1}{n}ES_\alpha(Y_1 + \ldots Y_n) < \frac{1}{m}ES_\alpha(Y_1 + \ldots + Y_m)$$

If $Y \sim N(\mu, \sigma^2)$ then the quantile of order α, denoted by q_α is given by

$$q_\alpha = \mu + \sigma\Phi^{-1}(\alpha)$$

and since $VaR_\alpha = -q_\alpha$ it follows that

$$VaR_\alpha(Y) = -\mu - \sigma\Phi^{-1}(\alpha)$$

Since $ES_\alpha = \frac{1}{\alpha}\int_0^\alpha VaR_u du$ a direct calculus shows that

$$ES_\alpha(Y) = -\mu + \frac{\sigma}{\alpha}\varphi[\Phi^{-1}(\alpha)]$$

Consider now that $Y_i \overset{i.i.d.}{\sim} F$ for all positive integers i, such that $E(Y_i) = \mu$ and

$var(Y_i) = \sigma^2$. Then by central limit theorem, for large enough n, $Y_1 + \ldots Y_n$ is approximately distributed with $N(n\mu, n\sigma^2)$. Hence, for $m \leq n$ and m large enough and $\alpha \in (0, 0.5)$,

$$\begin{aligned}
\frac{1}{n}VaR_\alpha(Y_1 + \ldots Y_n) &\leq \frac{1}{m}VaR_\alpha(Y_1 + \ldots Y_m) \\
\frac{1}{n}\left[-n\mu - \sqrt{n}\sigma\Phi^{-1}(\alpha)\right] &\leq \frac{1}{m}\left[-m\mu - \sqrt{m}\sigma\Phi^{-1}(\alpha)\right] \\
-\mu - \frac{1}{\sqrt{n}}\sigma\Phi^{-1}(\alpha) &\leq -\mu - \frac{1}{\sqrt{m}}\sigma\Phi^{-1}(\alpha) \\
\frac{1}{\sqrt{n}} &\leq \frac{1}{\sqrt{m}}
\end{aligned}$$

Remark that for $\alpha \in (0, 0.5)$ we know that $\Phi^{-1}(\alpha) < 0$. Hence the inequality is reversed for $\alpha > 0.5$ but this is not practically relevant for risk management.

Similarly, the monotonicity of specific risk condition is equivalent for ES to show that for $m \leq n$

$$\begin{aligned}
\frac{1}{n}ES_\alpha(Y_1 + \ldots Y_n) &\leq \frac{1}{m}ES_\alpha(Y_1 + \ldots Y_m) \\
\frac{1}{n}\left[-n\mu + \sqrt{n}\frac{\sigma}{\alpha}\varphi[\Phi^{-1}(\alpha)]\right] &\leq \frac{1}{m}\left[-m\mu + \sqrt{m}\frac{\sigma}{\alpha}\varphi[\Phi^{-1}(\alpha)]\right] \\
-\mu + \frac{1}{\sqrt{n}}\frac{\sigma}{\alpha}\varphi[\Phi^{-1}(\alpha)] &\leq -\mu + \frac{1}{\sqrt{m}}\frac{\sigma}{\alpha}\varphi[\Phi^{-1}(\alpha)] \\
\frac{1}{\sqrt{n}} &\leq \frac{1}{\sqrt{m}}
\end{aligned}$$

where the last inequality follows because $\varphi[\Phi^{-1}(\alpha)]$ is always positive.

We shall prove now that the same condition is satisfied not necessarily asymptotically, and also not only for the gaussian case. Consider that $Y \sim Cauchy(\mu, \gamma)$. The family of Cauchy distributions is closed to convolution , that is if $Y_1 \sim Cauchy(\mu_1, \gamma_1)$ and $Y_2 \sim Cauchy(\mu_2, \gamma_2)$ then $Y_1 + Y_2 \sim Cauchy(\mu_1 + \mu_2, \gamma_1 + \gamma_2)$. The cdf of $Y \sim Cauchy(\mu, \gamma)$ is given by $F_Y(y) = \frac{1}{\pi}\arctan\left(\frac{y-\mu}{\gamma}\right) + 0.5$ and the quantile function is $Q_\alpha = \mu + \gamma \tan(\pi(\alpha - 0.5))$. Then

$$VaR_\alpha(Y) = -\mu - \gamma \tan(\pi(\alpha - 0.5))$$

This formula allows also to derive the analytical expression for the ES of a Cauchy distributed variable.

$$\begin{aligned} ES_\alpha(Y) &= \frac{1}{\alpha}\int_0^\alpha VaR_u du \\ &= \frac{1}{\alpha}\int_0^\alpha [-\mu - \gamma \tan(\pi(u - 0.5))]du \\ &= -\mu + \frac{\gamma}{\alpha\pi}\ln|\cos\pi\left(\frac{1}{2}-\alpha\right)| \end{aligned}$$

Thus, for $\alpha \in (0, 0.5)$

$$ES_\alpha(Y) = -\mu + \frac{\gamma}{\alpha\pi}\ln\cos\pi\left(\frac{1}{2}-\alpha\right)$$

Therefore,

$$\begin{aligned} \frac{1}{n}VaR_\alpha(Y_1 + \dots Y_n) &\leq \frac{1}{m}VaR_\alpha(Y_1 + \dots Y_m) \\ \frac{1}{n}\left[-n\mu + n\gamma\tan\pi\left(\frac{1}{2}-\alpha\right)\right] &\leq \frac{1}{m}\left[-m\mu + m\gamma\tan\pi\left(\frac{1}{2}-\alpha\right)\right] \\ -\mu + \gamma\tan\pi\left(\frac{1}{2}-\alpha\right) &\leq -\mu + \gamma\tan\pi\left(\frac{1}{2}-\alpha\right) \end{aligned}$$

and

$$\begin{aligned} \frac{1}{n}ES_\alpha(Y_1 + \dots Y_n) &\leq \frac{1}{m}ES_\alpha(Y_1 + \dots Y_m) \\ \frac{1}{n}\left[-n\mu + n\frac{\gamma}{\alpha\pi}\ln\cos\pi\left(\frac{1}{2}-\alpha\right)\right] &\leq \frac{1}{m}\left[-m\mu + m\frac{\gamma}{\alpha\pi}\ln\cos\pi\left(\frac{1}{2}-\alpha\right)\right] \\ -\mu + \frac{\gamma}{\alpha\pi}\ln\cos\pi\left(\frac{1}{2}-\alpha\right) &\leq -\mu + \frac{\gamma}{\alpha\pi}\ln\cos\pi\left(\frac{1}{2}-\alpha\right) \end{aligned}$$

Thus, for Cauchy distributed payoffs, the inequalities are satisfied with equality, for both VaR and ES. Hence, in this case there is no gain in increasing the number of trades of the same kind.

The Procedure of Calculating the P-values for Unexpected Losses Measure

Assume that we have n order statistics, $Y_{[1]}, \dots, Y_{[n]}$, which represent our ordered financial returns. We define VaR and MS, respectively, as a monotonic transformation of the order statistics $Y_{[v]}$ and $Y_{[m]}$, with $v > m$. We want to calculate the probability $P_F(Y_{[v]} - Y_{[m]} \leq d)$ i.e. the probability that the difference between the two quantiles for VaR and MS is lower or equal to a certain value d. We impose d to be the difference between the estimated quantiles, namely the quantiles obtained from the empirical distribution of financial returns. If $Y_{[m]} = z$ and $Y_{[v]} = y$, the density function of $D = y - z$ is:

$$q(d) = K \int_{-\infty}^{\infty} F^{m-1}(z) \left[F(z+d) - F(z)\right]^{v-m-1} \left[1 - F(z+d)\right]^{n-v} f(z) f(z+d) dz \tag{25}$$

with $K = \frac{n!}{(m-1)!(v-m-1)!(n-v)!}$. The order statistics $Y_{[1]}, \dots, Y_{[n]}$ in a sample from any absolutely continuous distribution with cdf F can be transformed by the order-preserving probability integral transformation $\tilde{y} = F(y)$ into order statistics drawn from a uniform distribution on the interval $[0,1]$, $U_{[1]}, \dots, U_{[n]}$. Thus we can transform z and y into $\tilde{z} = F(z)$ and $\tilde{y} = F(y)$ and we can denote by $\tilde{D} = \tilde{y} - \tilde{z}$. Recalling the expressions for the density function and the distribution function of a uniform random variable in $[0,1]$, since $f(\tilde{z} + \tilde{d}) = 0$ for $\tilde{z} > 1 - \tilde{d}$, from formula (25) we have:

$$\begin{aligned} q(\tilde{d}) =& K \int_0^{\tilde{d}} \tilde{z}^{m-1} \tilde{d}^{v-m-1} \left(1 - \tilde{z} - \tilde{d}\right)^{n-v} d\tilde{z} \\ 0 \leq \tilde{d} \leq 1 \end{aligned} \tag{26}$$

This density function is absolutely equivalent to the density function in formula (25). Setting $\tilde{z} = \nu\left(1-\tilde{d}\right)$, formula (26) can be rewritten as:

$$\begin{aligned}
q(\tilde{d}) =& K\int_0^1 \nu^{m-1}(1-\tilde{d})^{m-1}\tilde{d}^{v-m-1}\left[(1-\nu)\left(1-\tilde{d}\right)\right]^{n-v}\left(1-\tilde{d}\right)d\nu \\
=& K(1-\tilde{d})^{m-1}\tilde{d}^{v-m-1}\left(1-\tilde{d}\right)\int_0^1 \nu^{m-1}\left[(1-\nu)\left(1-\tilde{d}\right)\right]^{n-v} d\nu \\
=& \frac{n!}{(m-1)!(v-m-1)!(n-v)!}(1-\tilde{d})^{n-v+m-1+1}\tilde{d}^{v-m-1}\underbrace{\int_0^1 \nu^{m-1}(1-\nu)^{n-\nu}d\nu}_{B(m,n-v-1)} \\
=& \frac{n!(m-1)!(n-v+1-1)!}{(m-1)!(v-m-1)!(n-v)!(n-v+m+1-1)!}(1-\tilde{d})^{n-v+m}\tilde{d}^{v-m-1} \\
=& \frac{1}{B(v-m,n-v+m+1)}(1-\tilde{d})^{n-v+m}\tilde{d}^{v-m-1} \\
& 0 \leq \tilde{d} \leq 1.
\end{aligned} \tag{27}$$

As a last step we compute the probability $P_F(D \leq d) = P_F(\tilde{D} \leq \tilde{d})$ as follows:

$$\begin{aligned}
P_F(D \leq d) =& \frac{1}{B(v-m,n-v+m+1)}\int_0^{\tilde{d}}(1-u)^{n-v+m}u^{v-m-1}du \\
=& \mathcal{B}_{\tilde{d}}(v-m,n-v+m+1) \\
& 0 \leq \tilde{d} \leq 1.
\end{aligned} \tag{28}$$

Formula 28 is exactly the closed-form expression we use to calculate the p-values.

Further Results Capital Requirements Calculations

Figure 9 show the two-week losses, the expected losses (computed according to the nonparametric VaR), the unexpected losses (calculated on the basis of the nonparametric MS), the unexpected losses adjusted for parameter risk estimation (i.e. the upper part of the 95% confidence interval for the nonparametric MS) and the capital requirements computed according to BCBS regulation ignoring stressed scenarios (first term of formula (23)) for an investment of \$100 in the S&P 500 and in the

USD/GBP exchange rate. In Figure 9 the unexpected losses adjusted for parameter risk estimation are obtained as the upper part of the 95% parametric confidence interval for the nonparametric MS. This parametric confidence interval and the BCBS capital requirements are calculated assuming a gaussian distribution for the returns of the asset. Figure 9 gives a further confirmation of the inadequacy of the gaussian method since all our measures of expected losses and unexpected losses (with and without adjustment for parameter risk estimation) are frequently exceeded by the realized losses.

[Figure 9 about here.]

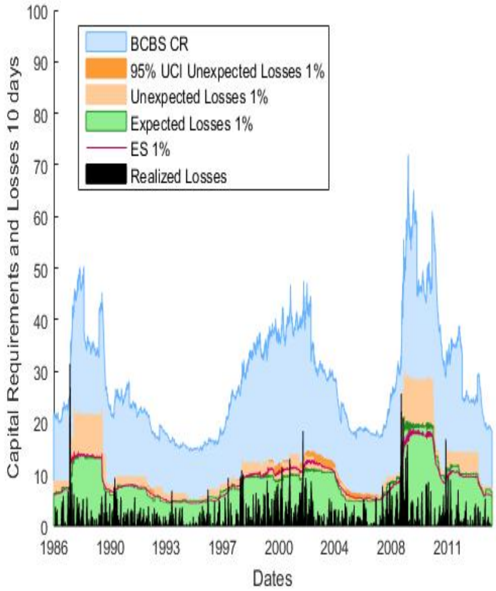
Using the BCBS capital requirements levels is not only inefficient but it may also cause confusion when comparing different models. For example, for the gaussian model the BCBS capital requirements are much lower than those obtained under the nonparametric or the NIG models. This may induce a false inference favoring the gaussian model over the other approaches while clearly the gaussian model is inapt to capture extreme tail losses, as demonstrated in this paper in earlier sections.

The results presented in Figure 10 indicate that although GARCH models react quicker to changes in market risk, the difference between the minimum capital requirements calculated based on the first term of formula (23) and the estimated of risk given by those models is again extremely large over the entire period of investigation. Furthermore, while there is some improvement by employing a GJR GARCH model in the sense that the VaR estimates look closer to the realised losses for the S&P500 market, there seems to be very little improvement on the USD/GBP market. In addition, the entire set of calculations presented in Figure 10 is a lot more irregular, more spikes, than the results presented in the paper under the NIG model, nonparametric model and the gaussian model. This implies that the frictions costs generated in order to satisfy risk management regulations is a lot higher. Banks would need to top-up their buffer account with large sums of money and soon after they will have to decrease their reserves. This suggests that GARCH models could be very capital intensive and somehow unnecessarily.

[Figure 10 about here.]

Figure 9: Time series of two-week losses, expected losses computed according to the nonparametric VaR, unexpected losses calculated on the basis of the nonparametric MS, upper part of the 95% parametric confidence interval for the nonparametric MS, the parametric ES, capital requirements computed according to BCBS (only first term in formula (23)) for the parametric VaR, for an investment of \$100 in the S&P 500 and in the USD/GBP exchange rate. The 95% parametric confidence interval for the nonparametric MS, the parametric ES and the BCBS capital requirements are calculated assuming a gaussian distribution for the returns of the asset.

(a) S&P 500 under gaussian



(b) USD/GBP under gaussian

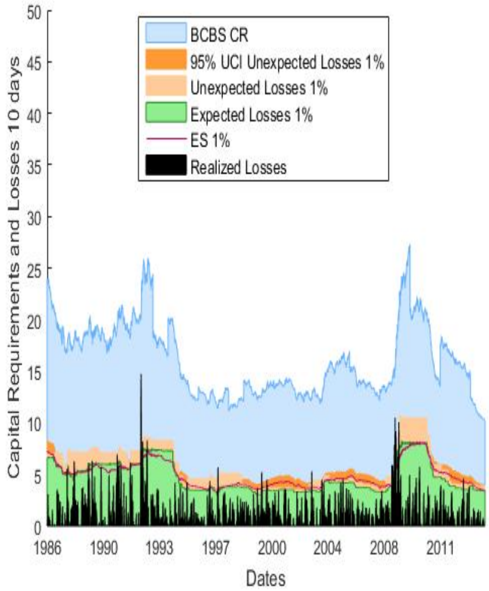
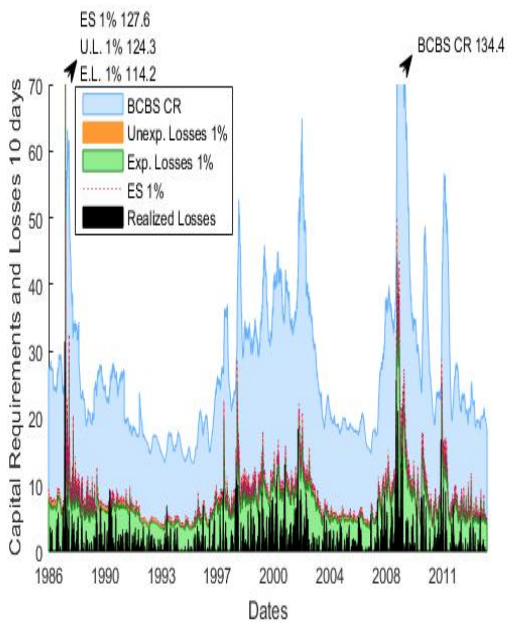
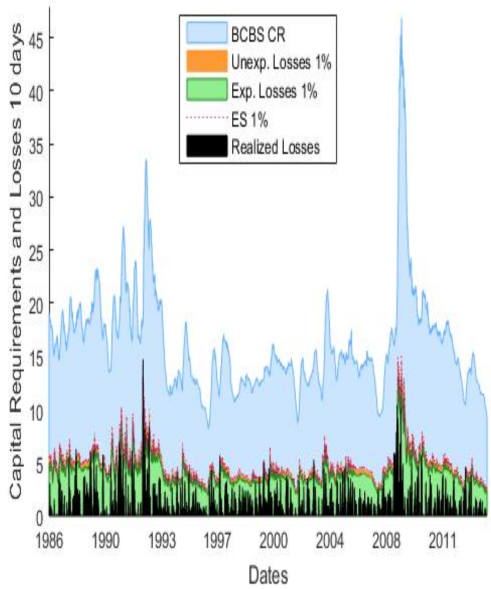


Figure 10: Time series of two-week losses, parametric VaR, parametric ES, parametric MS and capital requirements computed according to BCBS for the parametric VaR, for an investment of \$100 in the S&P 500 and in the USD/GBP exchange rate. The parametric VaR, the parametric ES, the parametric MS and the BCBS capital requirements (only first term in formula (23)) are calculated assuming a GARCH(1,1) model with Normal innovations and a GJR-GARCH(1,1) model with Normal innovations for the for the returns of the asset. The two plots concerning the S&P 500 have been cut and the relevant peaks are displayed next to the arrow.

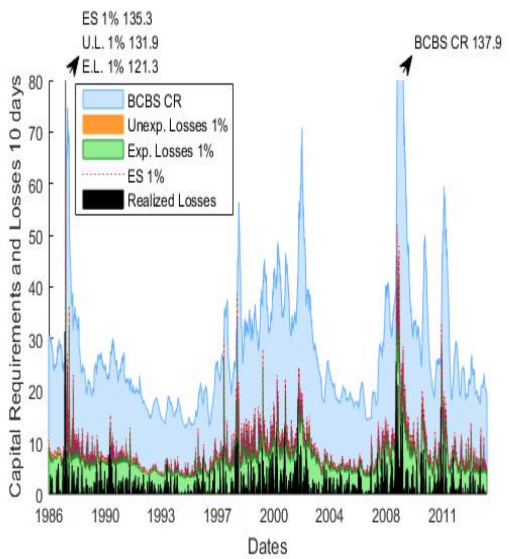
(a) S&P 500 under GARCH(1,1)



(b) USD/GBP under GARCH(1,1)



(a) S&P 500 under GJR-GARCH(1,1)



(b) USD/GBP under GJR-GARCH(1,1)